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MATHEMATICAL AND NUMERICAL MODELING OF THE COUPLED DYNAMIC THERMOELASTIC PROBLEMS FOR ISOTROPIC BODIES

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Abstract. A statement of the two-dimensional coupled thermodynamic boundary problem for isotropic bodies is presented in this paper. Corresponding explicit and implicit finite difference schemes are developed. Obtained schemes are solved by means of elimination method and recurrence formulas, respectively. A comparative solution clearly displays a good coincidence.

Keywords: thermo-elasticity, coupled problem, thermal conductivity, difference equations, explicit scheme, implicit scheme, grid method, elimination method.

AMS Subject Classification: 74S20.

1. Introduction

The investigation of deformation process in elastic and plastic bodies, taking into account temperature distribution plays an important role in many applications of scientific and engineering problems associated with heating various structures and its elements under thermomechanical load.

The aforementioned thermomechanical deformation process of solids may be described by coupled and uncoupled thermoelastic or thermoplastic boundary value problems.

Recently researchers have paid special attention to adequate mathematical modeling of coupled thermomechanical eformation processes in isotropic and anisotropic materials. The coupled thermodynamic problem was firstly considered by Biot [2] in 1956.

Further, those investigations were continued in many papers, among them the most popular are [1, 3, 5, 6, 9], and others.

In general, thermodynamic linear coupled boundary value problem for elastic bodies consists of the motion equations in conjunction with the heat equation.

Coupled thermoelastic boundary problem, generally consisting of three hyperbolic and a parabolic heat equation depending on three components of the displacement vector and temperature, is very complexed. It may be solved analytically only for some single dimensional particular cases or for specially selected shapes of solids and coordinate systems. This paper deals with numerical solution of two-dimensional dynamical coupled problems of thermoelasticity for isotropic bodies.

Using finite difference methods, explicit and implicit schemes were constructed. For numerical solution of finite difference equations the elimination method and recurrence formulas (in the case of explicit schemes), were applied. Comparison in terms of numerical results and graphs, of the two methods proves reliability and validity of the obtained results.

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Problem 1.1. The coupled thermodynamic thermoelastic boundary value problem consists of the system of equations and equalities, including motion equations for isotropic materials [7]

$$\sigma_{ij,j} + X_i = \rho \ddot{u}_i. \tag{1}$$

Duhamel-Neuman thermoelasticity constitutive relations are

$$\sigma_{ij} = \lambda\theta\delta_{ij} + \mu\varepsilon_{ij} - \alpha(3\lambda + 2\mu)(T - T_0)\delta_{ij}. \tag{2}$$

Cauchy equalities are

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \tag{3}$$

for the heat equation for isotropic bodies

$$\lambda_0 T_{,ii} - c_\varepsilon \dot{T} - \alpha(3\lambda + 2\mu)T_0\dot{\varepsilon}_{ii} = 0, \tag{4}$$

with corresponding initial

$$u_i|_{t=t_0} = \varphi_i,\, \dot{u}_i|_{t=t_0} = \psi_i,\, T|_{t=t_0} = T_0, \tag{5}$$

and boundary conditions

$$u_i|_{\sum_1} = u_i^0,\, T|_{\sum} = \bar{T}_0,\, \sigma_{ij}n_j|_{\sum_2} = S_i^0, \tag{6}$$

where, $c_\varepsilon-$ denotes heat at a constant deformation, $\alpha-$ corresponds to thermal expansion coefficient, λ_0- is the heat flow coefficient, $\sigma_{ij}-$ stress tensor, $\varepsilon_{ij}-$ strain tensor, u_i- displacement, $T-$ temperature, T_0- initial temperature X_i- volume force, $\lambda, \mu-$ Lame constants, $\theta-$ spherical part of strain tensor, $\rho-$ density of the body, $\delta_{ij}-$ delta Kronecker symbol.
Equations (1-6) are applied for the two dimensional cases.

Substituting eq.(3) into eq.(2) and obtained results into eq.(1) we get the equation of motion for displacement

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + (\lambda + \mu)\frac{\partial^2 v}{\partial x \partial y} - (3\lambda + 2\mu)\alpha\frac{\partial T}{\partial x} = \rho\frac{\partial^2 u}{\partial t^2}, \tag{7}$$

$$(\lambda + 2\mu)\frac{\partial^2 v}{\partial y^2} + \mu\frac{\partial^2 v}{\partial x^2} + (\lambda + \mu)\frac{\partial^2 u}{\partial x \partial y} - (3\lambda + 2\mu)\alpha\frac{\partial T}{\partial y} = \rho\frac{\partial^2 v}{\partial t^2}, \tag{8}$$

and 2D heat equations

$$\lambda_0(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) - c_\varepsilon\frac{\partial T}{\partial t} - \alpha(3\lambda + 2\mu)T_0(\frac{\partial^2 u}{\partial x \partial t} + \frac{\partial^2 v}{\partial y \partial t}) = 0, \tag{9}$$

with initial

$$u\,(x, y, t)|_{t=0} = \phi_1, \quad \left.\frac{\partial u}{\partial t}\right|_{t=0} = \psi_1, \quad v\,(x, y, t)|_{t=0} = \phi_2, \quad \left.\frac{\partial v}{\partial t}\right|_{t=0} = \psi_2, \quad T\,(x, y, t)|_{t=0} = T_0,$$

and boundary conditions in 2D case

$$u\,(x,\, y, t)|_{x=0} = u_0, \quad u\,(x, y,\, t)|_{x=\ell_1} = \bar{u}_0, \quad u\,(x,\, y, t)|_{y=0} = u_0', \quad u\,(x, y,\, t)|_{y=\ell\,2} = \bar{u}_0',$$

$$v\,(x,\, y, t)|_{x=0} = v_0, \quad v\,(x, y,\, t)|_{x=\ell\,1} = \bar{v}_0, \quad v\,(x,\, y, t)|_{y=0} = v_0', \quad v\,(x, y,\, t)|_{y=\ell\,2} = \bar{v}_0',$$

$$T\,(x, y, t)|_{x=0} = T_1\,(t)\,, \quad T\,(x, y, t)|_{x=\ell\,1} = T_2\,(t)\,,$$

$$T\,(x, y, t)|_{y=0} = {T_1}'\,(t)\,, \quad T\,(x, y, t)|_{y=\ell\,2} = T_2'\,(t)\,.$$

2. Finite difference equations

Considering within domain $t \geq 0$, $0 \leq x \leq l_1$, $0 \leq y \leq l_2$ two sets of parallel lines $x = ih_1$ $(i = \overline{0,n})$, $y = jh_2$ $(j = \overline{0,n})$, $t = k\tau$ $(k = 0,1,2,...)$ and replacing corresponding derivatives in eqs.(7-9) by difference equivalents, we obtain [8]

$$\left.\begin{array}{l}
(\lambda+2\mu)\frac{u^k_{i+1,j}-2u^k_{i,j}+u^k_{i-1,j}}{h_1^2}+(\lambda+\mu)\frac{v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1}}{4h_1h_2}+\\
\mu\frac{u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1}}{h_2^2}-\gamma\frac{T^k_{i+1,j}-T^k_{i-1,j}}{2h_1}=\rho\frac{u^{k+1}_{i,j}-2u^k_{i,j}+u^{k-1}_{i,j}}{\tau^2}\\
(\lambda+2\mu)\frac{v^k_{i,j+1}-2v^k_{i,j}+v^k_{i,j-1}}{h_1^2}+(\lambda+\mu)\frac{u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1}}{4h_1h_2}+\\
\mu\frac{v^k_{i+1,j}-2v^k_{i,j}+v^k_{i-1,j}}{h_2^2}-\gamma\frac{T^k_{i,j+1}-T^k_{i,j-1}}{2h_2}=\rho\frac{v^{k+1}_{i,j}-2v^k_{i,j}+v^{k-1}_{i,j}}{\tau^2}
\end{array}\right\}, \tag{10}$$

and the heat equation

$$\begin{array}{l}
\lambda_0\left(\frac{T^k_{i+1,j}-2T^k_{i,j}+T^k_{i-1,j}}{h_1^2}+\frac{T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1}}{h_2^2}\right)-c_\varepsilon\frac{T^{k+1}_{i,j}-T^k_{i,j}}{\tau}-\\
-\gamma T_0\left(\frac{u^{k+1}_{i+1,j}-u^{k+1}_{i-1,j}-u^{k-1}_{i+1,j}+u^{k-1}_{i-1,j}}{4h_1\tau}+\frac{v^{k+1}_{i,j+1}-v^{k+1}_{i,j-1}-v^{k-1}_{i,j+1}+v^{k-1}_{i,j-1}}{4h_2\tau}\right)=0,
\end{array} \tag{11}$$

where $\gamma = \alpha(3\lambda + 2\mu)$, $n-$number of segments. Solving discreet system of equations (10) and (11) in terms of $u^{k+1}_{i,j}$, $v^{k+1}_{i,j}$, $T^{k+1}_{i,j}$ respectively, we get [4]

$$\begin{array}{l}
u^{k+1}_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^k_{i+1,j}-2u^k_{i,j}+u^k_{i-1,j}}{h_1^2}+\mu\frac{u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1}}{h_2^2}+\right.\\
\left.+(\lambda+\mu)\frac{v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1}}{4h_1h_2}-\gamma\,\frac{T^k_{i+1,j}-T^k_{i-1,j}}{2h_1}\right)+2u^k_{i,j}-u^{k-1}_{i,j},
\end{array} \tag{12}$$

$$\begin{array}{l}
v^{k+1}_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v^k_{i,j+1}-2v^k_{i,j}+u^k_{i,j-1}}{h_2^2}+\mu\frac{v^k_{i+1,j}-2v^k_{i,j}+v^k_{i-1,j}}{h_1^2}+\right.\\
\left.+(\lambda+\mu)\frac{u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1}}{4h_1h_2}-\gamma\,\frac{T^k_{i,j+1}-T^k_{i,j-1}}{2h_2}\right)+2v^k_{i,j}-v^{k-1}_{i,j}
\end{array} \tag{13}$$

$$\begin{array}{l}
T^{k+1}_{i,j}=\frac{\tau}{c_\varepsilon}\left(\lambda_0\left(\frac{T^k_{i+1,j}-2T^k_{i,j}+T^k_{i-1,j}}{h_1^2}+\frac{T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1}}{h_2^2}\right)-\right.\\
\left.-\gamma T_0\left(\frac{u^{k+1}_{i+1,j}-u^{k+1}_{i-1,j}-u^{k-1}_{i+1,j}+u^{k-1}_{i-1,j}}{4h_1\tau}+\frac{v^{k+1}_{i,j+1}-v^{k+1}_{i,j-1}-v^{k-1}_{i,j+1}+v^{k-1}_{i,j-1}}{4h_2\tau}\right)\right)+T^k_{i,j}.
\end{array} \tag{14}$$

As it can be seen, eq. (12,13) and eq. (14) allow us to find function values $u(x,y,t), v(x,y,t)$, $T(x,y,t)$ at the layer t^{k+1}using the given values of those functions at two aforetime layers. Using appropriate values of $u(x,y,t)$ and $v(x,y,t)$for two primary layers $k = 0$ and $k = 1$ we can obtain from the initial conditions

$$u^0_{i,j}=\varphi_1(x_i,y_j),\ \ v^0_{i,j}=\varphi_2(x_i,y_j),\ \ T^0_{i,j}=T_0.$$

Rewriting eq.(12) for k=0

$$\begin{array}{l}
u^1_{i,j}=\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j}}{h_1^2}+\mu\frac{u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1}}{h_2^2}+\right.\\
\left.+(\lambda+\mu)\frac{v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1}}{4h_1h_2}-\gamma\,\frac{T^0_{i+1,j}-T^0_{i-1,j}}{2h_1}\right)+2u^0_{i,j}-u^{-1}_{i,j},
\end{array} \tag{15}$$

and the initial condition $\frac{\partial u}{\partial t}\big|_{t=0} = \psi_1$ in the following form

$$\frac{u^1_{i,j}-u^{-1}_{i,j}}{2\tau}=\psi_1(x_i,y_j),$$

or

$$u^1_{i,j} = 2\tau\psi_1(x_i, y_j) + u^{-1}_{i,j}, \tag{16}$$

and eliminating $u^{-1}_{i,j}$ from eqs. (15,16) we postulate

$$u^1_{i,j} = \frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{u^0_{i+1,j}-2u^0_{i,j}+u^0_{i-1,j}}{h_1^2} + \mu\frac{u^0_{i,j+1}-2u^0_{i,j}+u^0_{i,j-1}}{h_2^2} + \right.\right.$$
$$\left.\left. +(\lambda+\mu)\frac{v^0_{i+1,j+1}-v^0_{i-1,j+1}-v^0_{i+1,j-1}+v^0_{i-1,j-1}}{4h_1h_2} - \gamma\ \frac{T^0_{i+1,j}-T^0_{i-1,j}}{2h_1}\right) + 2u^0_{i,j} + 2\tau\psi_1\right). \tag{17}$$

Exactly in the same way from eq.(13) we can derive the function $u(x, y, t)$

$$v^1_{i,j} = \frac{1}{2}\left(\frac{\tau^2}{\rho}\left((\lambda+2\mu)\frac{v^0_{i,j+1}-2v^0_{i,j}+u^0_{i,j-1}}{h_2^2} + \mu\frac{v^0_{i+1,j}-2v^0_{i,j}+v^0_{i-1,j}}{h_1^2} + \right.\right.$$
$$\left.\left. +(\lambda+\mu)\frac{u^0_{i+1,j+1}-u^0_{i-1,j+1}-u^0_{i+1,j-1}+u^0_{i-1,j-1}}{4h_1h_2} - \gamma\ \frac{T^0_{i,j+1}-T^0_{i,j-1}}{2h_2}\right) + 2v^0_{i,j} + 2\tau\psi_2\right). \tag{18}$$

Replacing mixed derivatives in eq.(11) with another finite difference we can deduce the relation for values of $T(x, y, t)$ at the first time layer

$$T^1_{i,j} = \frac{\tau}{c_\varepsilon}\left(\lambda_0\left(\frac{T^0_{i+1,j}-2T^0_{i,j}+T^0_{i-1,j}}{h_1^2} + \frac{T^0_{i,j+1}-2T^0_{i,j}+T^0_{i,j-1}}{h_2^2}\right) - \right.$$
$$\left. -\gamma T_0\left(\frac{u^1_{i+1,j}-u^1_{i-1,j}-u^0_{i+1,j}+u^0_{i-1,j}}{2h_1\tau} + \frac{v^1_{i,j+1}-v^1_{i,j-1}-v^0_{i,j+1}+v^0_{i,j-1}}{2h_2\tau}\right)\right) + T^0_{i,j}. \tag{19}$$

In the above-mentioned formulae, an explicit scheme has just been used. Now let us consider an implicit scheme for numerical solutions of coupled thermoelasticity problems. In this case, we transform eqs.(7-9) to the following form

$$(\lambda+2\mu)\frac{u^{k+1}_{i+1,j}-2u^{k+1}_{i,j}+u^{k+1}_{i-1,j}}{h_1^2} + (\lambda+\mu)\frac{v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1}}{4h_1h_2} +$$
$$\mu\frac{u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1}}{h_2^2} - \gamma\frac{T^k_{i+1,j}-T^k_{i-1,j}}{2h_1} = \rho\frac{u^{k+1}_{i,j}-2u^k_{i,j}+u^{k-1}_{i,j}}{\tau^2}, \tag{20}$$

$$(\lambda+2\mu)\frac{v^k_{i,j+1}-2v^k_{i,j}+v^k_{i,j-1}}{h_1^2} + (\lambda+\mu)\frac{u^k_{i+1,j+1}-u^k_{i-1,j+1}-u^k_{i+1,j-1}+u^k_{i-1,j-1}}{4h_1h_2} +$$
$$\mu\frac{v^{k+1}_{i+1,j}-2v^{k+1}_{i,j}+v^{k+1}_{i-1,j}}{h_2^2} - \gamma\frac{T^k_{i,j+1}-T^k_{i,j-1}}{2h_2} = \rho\frac{v^{k+1}_{i,j}-2v^k_{i,j}+v^{k-1}_{i,j}}{\tau^2}, \tag{21}$$

$$\lambda_0(\frac{T^{k+1}_{i+1,j}-2T^{k+1}_{i,j}+T^{k+1}_{i-1,j}}{h_1^2} + \frac{T^k_{i,j+1}-2T^k_{i,j}+T^k_{i,j-1}}{h_2^2}) - c_\varepsilon\frac{T^{k+1}_{i,j}-T^k_{i,j}}{\tau} -$$
$$-\gamma T_0\left(\frac{u^{k+1}_{i+1,j}-u^{k+1}_{i-1,j}-u^{k-1}_{i+1,j}+u^{k-1}_{i-1,j}}{4h_1\tau} + \frac{v^{k+1}_{i,j+1}-v^{k+1}_{i,j-1}-v^{k-1}_{i,j+1}+v^{k-1}_{i,j-1}}{4h_2\tau}\right) = 0. \tag{22}$$

The stencil of eq.(20) is three-point in space

$$a_i u^{k+1}_{i+1,j} + b_i u^{k+1}_{i,j} + c_i u^{k+1}_{i-1,j} = f_i, \tag{23}$$

where $a_i = \frac{\lambda+2\mu}{h_1^2}$, $b_i = -\frac{\rho}{\tau^2} - \frac{2(\lambda+2\mu)}{h_1^2}$, $c_i = \frac{\lambda+2\mu}{h_1^2}$,

$$f_i = \rho\frac{-2u^k_{i,j}+u^{k-1}_{i,j}}{\tau^2} - (\mu\frac{u^k_{i,j+1}-2u^k_{i,j}+u^k_{i,j-1}}{h_2^2} + (\lambda+\mu)\frac{v^k_{i+1,j+1}-v^k_{i-1,j+1}-v^k_{i+1,j-1}+v^k_{i-1,j-1}}{4h_1h_2} -$$
$$-\gamma\frac{T^k_{i+1,j}-T^k_{i-1,j}}{2h_1}).$$

The finite difference eq.(21) may also be reduced to the following form

$$a_i v^{k+1}_{i+1,j} + b_i v^{k+1}_{i,j} + c_i v^{k+1}_{i-1,j} = f_i, \tag{24}$$

where $a_i = \frac{\mu}{h_1^2}$, $b_i = -\frac{\rho}{\tau^2} - \frac{2\mu}{h_1^2}$, $c_i = \frac{\mu}{h_1^2}$,

$$f_i = \rho\frac{-2v_{i,j}^k + v_{i,j}^{k-1}}{\tau^2} - ((\lambda+2\mu)\frac{v_{i,j+1}^k - 2v_{i,j}^k + v_{i,j-1}^k}{h_1^2} + (\lambda+\mu)\frac{u_{i+1,j+1}^k - u_{i-1,j+1}^k - u_{i+1,j-1}^k + u_{i-1,j-1}^k}{4h_1h_2} - $$
$$-\gamma\frac{T_{i,j+1}^k - T_{i,j-1}^k}{2h_2}),$$

and the discreet heat eq.(22) takes following form

$$a_i T_{i+1,j}^{k+1} + b_i T_{i,j}^{k+1} + c_i T_{i-1,j}^{k+1} = f_i, \qquad (25)$$

where

$$a_i = \frac{\lambda_0}{h_1^2}, b_i = -\frac{2\lambda_0}{h_1^2} - \frac{c_\varepsilon}{\tau}, c_i = \frac{\lambda_0}{h_1^2},$$

$$f_0 = \gamma T_0\left(\frac{u_{i+1,j}^{k+1} - u_{i-1,j}^{k+1} - u_{i+1,j}^{k-1} + u_{i-1,j}^{k-1}}{4h_1\tau} + \frac{v_{i,j+1}^{k+1} - v_{i,j-1}^{k+1} - v_{i,j+1}^{k-1} + v_{i,j-1}^{k-1}}{4h_2\tau}\right) - c_\varepsilon\frac{T_{i,j}^k}{\tau} -$$
$$-\lambda_0\frac{T_{i,j+1}^k - 2T_{i,j}^k + T_{i,j-1}^k}{h_2^2}.$$

Values of unknown functions $u(x,y,t)$, $v(x,y,t)$ and $T(x,y,t)$ at the first two time layers, can be straightforwardly determined from initial conditions and formulae eqs.(17-19). Values of state variables subsequent layers are computed by solving eqs.(23-25) with the elimination method taking into account, given initial and boundary conditions.

3. Numerical tests

As a test case example coupled thermoelasticity problem (1-6) was simulated using the recurrence formulas and elimination method for the following constants, initial and boundary conditions:

$$\lambda_0 = 0.053, \ \ \lambda = 0.92, \ \ \mu = 0.48, \ \ \alpha = 0.064, c_\varepsilon = 3.47, \ \ T_0 = 25,$$
$$h_1 = 0.1, \ \ h_2 = 0.1, \ \ \tau = 0.01, \ \ \rho = 0.783, \ \ \ell_1 = \ell_2 = 1,$$
$$u\,(x,y,t)|_{t=0} = 0, \ \ v\,(x,y,t)|_{t=0} = 0, \ \ T\,(x,y,t)|_{t=0} = T_0 \sin(\pi x(i))\sin(\pi y(j)),$$
$$u\,(x,y,t)| = 0, \ \ v\,(x,y,t)| = 0, \ \ T\,(x,y,t)| = 0,$$

where Γ - boundary of the body.

The following Figures 1-6 show the displacement components $u(x,y,t)$, $v(x,y,t)$ and temperature $T(x,y,t)$ distributions in 2D space. It should be noted, that every unknown value was computed using the elimination method (green line) and recurrence formulas (red line).

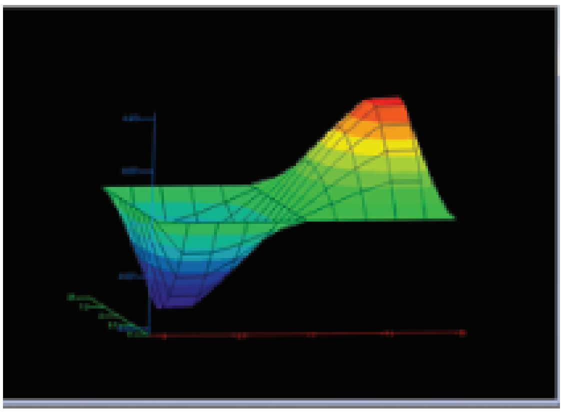

Figure 1. The displacement $u(x,y,t)$ along the x-y-axis at $t = 0.1$(recurrence formula).

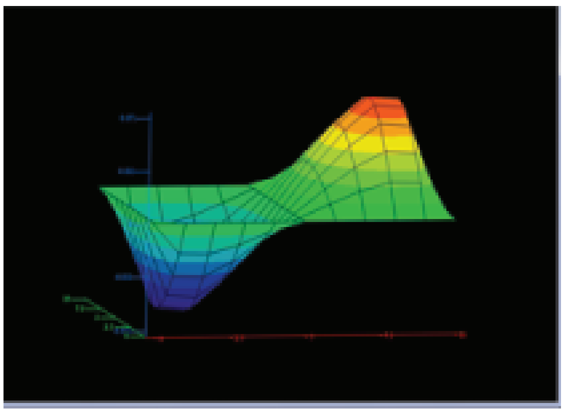

Figure 2. The displacement $u(x,y,t)$along the x-y-axis at $t = 0.1$(elimination method).

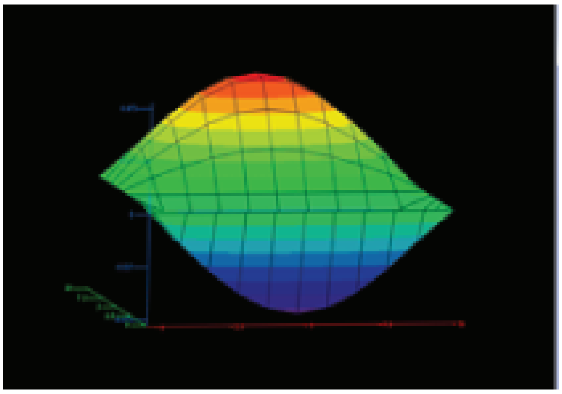

Figure 3. The displacement $v(x, y, t)$along the x-y-axis at $t = 0.1$(recurrence formula).

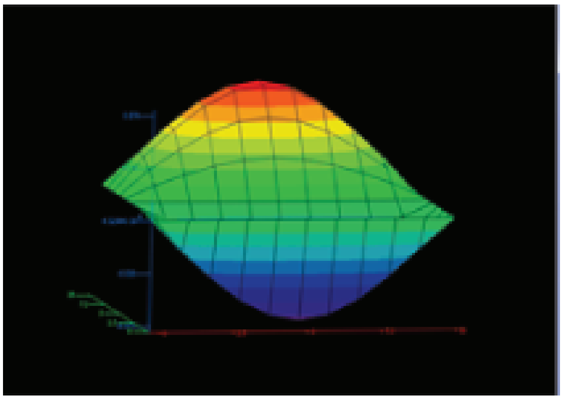

Figure 4. The displacement $v(x, y, t)$along the x-y-axis at $t = 0.1$(elimination method).

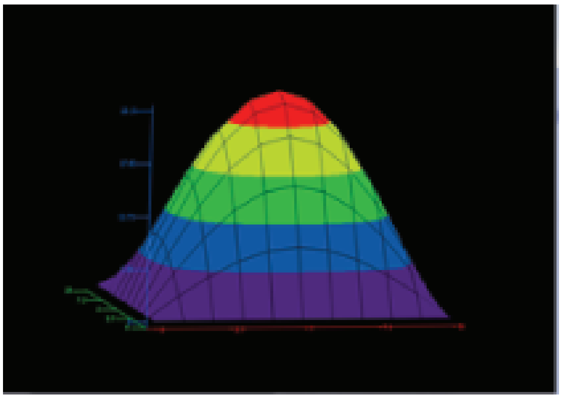

Figure 5. The displacement $T(x, y, t)$along the x-y-axis at $t = 0.1$(recurrence formula).

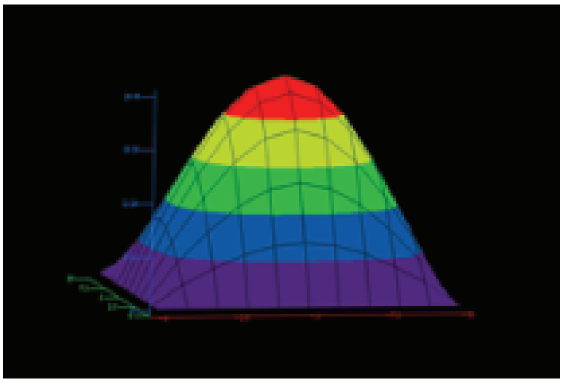

Figure 6. The displacement $T(x, y, t)$along the x-y-axis at $t = 0.1$(elimination method).

Table 1a. Values of the displacement component $u(x, y, t)$at time layer k=9 computed by elimination method.

x/y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	−0.0243	−0.0239	−0.0176	−0.0093
0.2	0	−0.0433	−0.0429	−0.0315	−0.0166
0.3	0	−0.0595	−0.0588	−0.0431	−0.0227
0.4	0	−0.0698	−0.0690	−0.0506	−0.0266
0.5	0	−0.0734	−0.0725	−0.0532	−0.0280

Table 1b. Values of the displacement component $u(x, y, t)$ at time layer k=9 obtained by recurrence formulae.

x/y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	−0.0226	−0.0231	−0.0173	−0.0092
0.2	0	−0.0403	−0.0415	−0.0311	−0.0164
0.3	0	−0.0555	−0.0572	−0.0428	−0.0226
0.4	0	−0.0654	−0.0673	−0.0504	−0.0266
0.5	0	−0.0689	−0.0709	−0.0531	−0.0281

Table 2a. Values of the displacement component $v(x,y,t)$at time layer k=9 computed by elimination method.

x/y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	−0.0243	−0.0433	−0.0595	−0.0698
0.2	0	−0.0239	−0.0429	−0.0588	−0.0690
0.3	0	−0.0176	−0.0315	−0.0431	−0.0506
0.4	0	−0.0093	−0.0166	−0.0227	−0.0266
0.5	0	0	0	0	0

Table 2b. Values of the displacement component $v(x,y,t)$ at time layer k=9 obtained by recurrence formulae.

x/y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	−0.0243	−0.0432	−0.0592	−0.0696
0.2	0	−0.0241	−0.0431	−0.0590	−0.0693
0.3	0	−0.0179	−0.0320	−0.0438	−0.0514
0.4	0	−0.0095	−0.0170	−0.0233	−0.0274
0.5	0	0	0	0	0

Table 3a. Temperature $T(x,y,t)$ at time layer k=9 computed by elimination method.

x/y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	2.3571	4.4646	6.1562	7.2508
0.2	0	4.4646	8.3440	11.4176	13.3887
0.3	0	6.1562	11.4176	15.5730	18.2324
0.4	0	7.2508	13.3888	18.2324	21.3299
0.5	0	7.6292	14.0664	19.1450	22.3921

Table 3b. Temperature $T(x,y,t)$ at time layer k=9 obtained by recurrence formulae.

x/y	0	0.1	0.2	0.3	0.4
0	0	0	0	0	0
0.1	0	2.3658	4.1703	5.6840	6.6749
0.2	0	4.4894	7.9596	10.8524	12.7438
0.3	0	6.3739	11.3082	15.4204	18.1083
0.4	0	7.6537	13.5848	18.5268	21.7562
0.5	0	8.1867	14.5362	19.8262	23.2823

Comparison of numerical values of displacement components and temperature obtained by the elimination method (Table 1a, Table 2a, Table 3a) and recurrence formulae (Table 1b, Table 2b, Table 3b) clearly demonstrates its tight coincidence.

4. Conclusions

For two-dimensional coupled thermodynamic boundary value problems, explicit and implicit finite difference schemes were constructed. Obtained explicit and implicit schemes were solved using recurrence formulae and elimination method, respectively. A comparison of obtained numerical results by those two methods shows a good coincidence. The above mentioned methods may be easily applied for numerical solution of three dimensional thermodynamic coupled problems for elastic and plastic bodies.

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