TWMS J. Pure Appl. Math., V.12, N.2, 2021, pp.199-208

TWO-STAGE PROCEDURE OF H_∞ - PARAMETERIZATION OF STABILIZING CONTROLLERS APPLIED TO QUADROTOR FLIGHT CONTROL

V.B. LARIN[1], A.A. TUNIK[2]

Abstract. The problem of $H_\infty-$ parameterization of stabilizing controllers for one class of nonholonomic systems is considered. It includes the flight control systems of a quadrotor with a certain structure, consisting of inner contours for stabilizing the module of the quadrotor velocity vector and outer contours for control of this vector direction. Therefore, it is proposed to apply the two-stage procedure of $H_\infty-$ parameterization of stabilizing controllers to each inner contour at the 1st stage and then to each outer contour at the 2nd stage. Simulation of quadrotor planar circular reference track following in the calm and disturbed atmosphere proves the viability of obtained results.

Keywords: nonholonomic systems, quadrotor fight control, parameterization of stabilizing controllers, static output feedback, $L_2 - gain$ restriction.

AMS Subject Classification: 15A24, 93B35, 93C73.

1. Introduction and problem statement

Recently the problem of quadrotor flight control is considered in two aspects: the quadrotor is considered as holonomic [5, 9, 10] and nonholonomic [7, 16] system. The advantages and disadvantages of these approaches are considered in [7, 16], where it is stated, that for the outdoor application the last one is preferable from the viewpoint of path planning. In this case, the desired path is based on the space-indexed reference track using only spatial coordinates of the waypoints (so-called "waypoint navigation"). In the case of the holonomic system, it is necessary to apply more complicated double indexed path planning based on space and time indices for each waypoint. It is obvious, that motion control of the nonholonomic vehicle requires stabilization of its velocity vector module and control of this vector direction. In [9, 10] the first problem of stabilization of the quadrotor speed was solved via LQR-approach. Then the first and the second problems were solved in [16] via the same approach. It is acceptable when it is possible to neglect the inertia of the quadrotor motors. However, it does not work in many practical cases [2, 7, 14], when taking into account the motor's inertia is mandatory. As far as the motor's rotation rate is practically unobserved, the synthesis of flight control, in this case, requires the application of some different methods. For the sake of control system simplicity, it is preferable to apply for control law synthesis the linear matrix inequality (LMI) approach [1, 4, 8, 17], because it creates control law as the simplest static output feedback (SOF). Following [16], the motion control system for each axis of the quadrotor in the nonholonomic case consists of two contours: the inner contour for velocity control and the outer contour for position control.

[1]Institute of Mechanics, National Academy of Sciences of Ukraine, Kiev, Ukraine
[2]National Aviation University, Kiev, Ukraine
e-mail: vblarin@gmail.com, anatolitunik@gmail.com
Manuscript received September 2019.

That is why we consider the two-stage procedure of synthesis: determination of SOF for inner contour, and then a determination of SOF for outer contour considering the closed-loop system with SOF, obtained at the 1st stage, as a new system for the 2nd stage.

In this paper instead of a well-known LMI approach, we used a parameterization of all stabilizing $H_\infty-$ static state-feedback (SSF) gains and their application to SOF design [6]. The advantage of this approach from our point of view is the simplicity of its application because the reiterative solution of the algebraic Riccati equation proposed in [6] is simpler than the LMI reiterative solution proposed in [8, 17]. Flight control simulation for some practical models of the quadrotor in the case of planar flight at the constant altitude proves the efficiency of the proposed method. Although SOF synthesis for control of the quadrotor motion along each axis was made for the linearized models, the simulation of the planar (circular) motion, involving control systems for longitudinal and lateral axes simultaneously, was performed with all non-linearities, which are immanent for the real system. The results of this simulation confirm the viability of the proposed method.

2. Problem statement and algorithm for its solving

Consider the standard problem of the static output feedback (SOF) synthesis, which guarantees $H_\infty-$the rejection of external disturbances [1, 4, 8, 17]. These control laws were successfully applied to the flight control synthesis of various types of aerial vehicles [8, 17] and here it will be illustrated by application to the quadrotor flight control. Previously the procedures of synthesis were based on the application of LMI solution [1, 4, 8, 17]. Further development of this approach produced a new result based on the solution of the algebraic Riccati equation (ARE) [6]. In this case, we can create the same control strategy (SOF) with the same suppression of the external disturbances $H_\infty-$ norms. In the case of the quadrotor, we can formulate this problem as follows.

As it was stated in [5, 9] the quadrotor flight control problem can be considered based on the separation of its motions along 3 axes (longitudinal X, lateral Y, and vertical Z), taking into account the symmetry of construction of this aerial vehicle about these axes.

Let the linearized mathematical model of the quadrotor, describing its linear motion along the lateral axis Y, can be written in the following standard form [1, 8, 17]:

$$\frac{d\chi}{dt}(t) = A\chi(t) + B_u u(t) + B_d d(t), \upsilon(t) = C\chi(t), z(t) = \begin{bmatrix} \sqrt{Q} & 0 \\ 0 & \sqrt{R} \end{bmatrix} \begin{bmatrix} \chi(t) \\ u(t) \end{bmatrix}. \quad (1)$$

where χ,υ, and $\boldsymbol{z}$ are the state, output, and desired output vectors respectively. These vectors χ and υ include the following components:

$$\chi = [\chi_1, \chi_2, \chi_3, \chi_4, \chi_5]^T = [y, dy/dt, \phi, p, \Delta\Omega]^T, \ \ \upsilon = [y, dy/dt, \phi, p]^T. \quad (2)$$

In expression (2) y, dy/dt are a position of a quadrotor on the Y-axis, and its linear velocity along this axis, respectively; ϕ, p are roll angle and roll rate, and $\Delta\Omega$ is the increment of the rotation rate of quadrotor' motors, located on the Y-axis and produced by a control input $u(t)$. Also, it is necessary to mention that $d(t)$ is the exogenous disturbance, which is the component of the turbulent wind velocity acting along the Y-axis. The numerical values of matrices $A \in R^{5\times5}, B_u \in R^{5\times1}, B_d \in R^{5\times1}, C \in R^{4\times5}$ will be given later in the Case Study section. Eventually, it is necessary to note that Q, R are the weighting matrices defining the contributions of the state and control vectors in the desired output z. Their numerical values will be selected during the procedure of synthesis. Taking into account the similarity of the model of quadrotor linear motion about the X-axis we do not consider this case for the sake of brevity.

The final result of the control law synthesis for the model (1), based on the static output feedback (SOF) approach [1, 4, 8, 17], looks like:

$$u\,(t) = -K\upsilon\,(t) = -KC\,\chi(t), \tag{3}$$

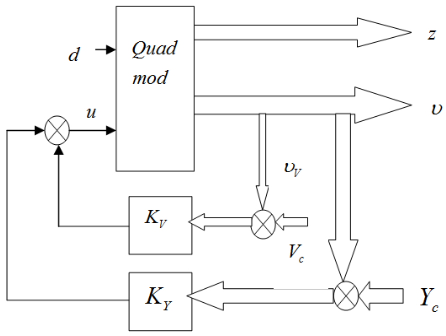

Figure 1. Scheme of the Y-axis control system.

where: $K \in R^{1\times4}, C \in R^{4\times5}$ K is the numerical feedback gain matrix. Now it is possible to formulate the standard problem of suppressing external disturbance$d$ by bounded L_2- gain [1, 4, 8, 17] using gain matrix K. Let us have a well-known integral-quadratic performance index:

$$J = \int_0^\infty \|z\ (t)\|^2 = \int_0^\infty \left(\chi^T Q x + u^T R u\right)\, dt, \tag{4}$$

which defines desired output signal $z(t)$in the system (1).
Then we define bounded system L_2 gain as follows [1, 4, 8, 17]:

$$\frac{\int_0^\infty \|z\ (t)\|^2\, dt}{\int_0^\infty \|d\ (t)\|^2\, dt} = \frac{\int_0^\infty \left(\chi^T Q\,\chi + u^T R u\right)\, dt}{\int_0^\infty (d^T d)\, dt} \le \gamma^2. \tag{5}$$

The problem consists of finding such SOF-gain $K \in R^{1\times4}$that satisfies inequality (5) for a closed-loop system. Note, that the minimal possible value of $\gamma$is denoted as γ^*. The gain matrix K in (3) used to be found by LMI – approach (see, for instance [1, 4, 8, 17], where further references are quoted).

Let us interpret this control problem when the quadrotor is considered as a nonholonomic system. As it was shown in [16], such a control system, represented in Fig.1, might be considered consisting of two loops: the inner loop for velocity control with command vector V_c and the outer loop for suppressing the deviation of quadrotor position y on the lateral axis from the command vector Y_c. The state χ_V, output υ_V, and command V_c vectors for the inner loop have the following forms respectively:

$$\chi_V = [dy/dt, \phi, p, \Delta\Omega]^T,$$

$$\upsilon_V = [dy/dt, \phi, p]^T,\ V_c = [v_c, 0, 0]^T\,, \tag{6}$$

where v_c is the velocity command signal. The state and output vectors for the outer loop are the same as in the system (2), meanwhile, the command vector looks like this:

$$Y_c = [y_c, 0, 0, 0]^T \,, \tag{7}$$

where y_c is the command signal. Command signals appearing in (6) and (7) are generated by reference track program. Note, that the two-loop structure of quadrotor flight control was proposed also in [2].

Taking into account the control structure in Fig.1, it is possible to solve the problem (1-5) for the quadrotor nonholonomic case by the consecutive two-stage procedure. The 1st stage consists of the problem (1-5) solution for the inner loop with state and output vectors defined by (6) and matrices $A^V \in R^{4\times4}, B_u^V \in R^{4\times1}, B_d^V \in R^{4\times1}, C^V \in R^{3\times1}, Q^V \in R^{4\times4}$in the system (1), truncated correspondingly with vectors (6). The solution of the problem (1-5) results in finding inner loop feedback gain matrix

$$K_V = [K_{dy/dt}^V, K_\phi^V, K_p^V]. \tag{8}$$

At the 2nd stage, the gain matrix (8) is used for the creation of the state-space model of the closed-loop outer contour with matrices$A^{out} \in R^{5\times5}, B_u^{out} \in R^{5\times1}, B_d^{out} = B_d, C \in R^{4\times5}$, which includes inner contour with feedback gains (8). Then we apply to this partially closed-loop system the procedure of synthesis based on the LMI-approach for finding feedback gain matrix for the outer contour:

$$K_Y = [K_y, K_{dy/dt}, K_\phi, K_p], \tag{9}$$

defining static output feedback for outer contour:

$$u\,(t) = -K_Y\,\upsilon\,(t) = -K_Y C\,\chi(t).$$

It is shown in [8, 17], that solution of problem (1-5) via LMI-approach requires multiple iterative solutions of LMI system for a given problem to find SOF matrix gain Kin (3) satisfying (5). From our point of view, this computational procedure is rather complicated. In [6] the same problem solving was made via iterative solutions of the algebraic Riccati equation to find parameterization of all static state feedback (SSF) $H_\infty-$ gains and then its application to the SOF – design. That is why it is possible to find solutions of the algebraic Riccati equations (ARE) instead of the LMI solution, which essentially alleviates computational problems. This statement can be summarized in the following theorem borrowed from [6]:
Consider a specified matrix $Q \geq 0$ such that $(A, Q^{1/2})$ is detectable, (A, B_u) is stabilizable, and a specified value $\gamma > \gamma^*$. Then there exists a SOF gain K such that $A_0 \equiv (A - B_u KC)$ is asymptotically stable with bounded L_2 gain, if and only if there exists a parameter matrix L such that

$$KC = R^{-1}(B_u^T P + L), \tag{10}$$

where is a solution to *ARE [10]*:

$$PA + A^T P + Q + \frac{1}{\gamma^2} P B_d B_d^T P - P B_u R^{-1} B_u^T P + L^T R^{-1} L = 0.$$

Let K_0 will be SSF-gain obtained after application of the LQR-procedure to a system (1) with given matrices in Q, R (3) assuming that all state variables are measured, i.e. $C = I_{5\times5}$ in our case, this SSF-gain has size $K_0 \in R^{1\times 5}$. On the other hand, in our case, SOF –gain has the size $K \in R^{1\times 4}$ (see (3)), and from (10) it follows:

$$K = R^{-1}(B_u^T P + L)C^T(C \times C^T)^{-1}. \tag{11}$$

It is proposed in [10] to use the singular value decomposition (SVD) of the matrix C:$C = USV^T = U[S_0 0]\begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$: for the pseudo-inverse matrix $C^+ = C^T(CC^T)^{-1}$ and the SOF-gain K calculation. Using this SVD, we obtain [6]:

$$C^+ = V_1(S_0)^{-1}U^T. \tag{12}$$

To recalculate the gain matrix K in the gain matrix F it is proposed [6] to use the projection onto null space perpendicular C, using the matrix

$$f = I - V_2V_2^T. \tag{13}$$

After these preliminary remarks, we will present the algorithm proposed in [6], for the solution of the aforementioned problem of control system design.

3. Algorithm for quadrotor flight control synthesis borrowed from [10]

Define: Matrices of quadrotor state-space model: A, B_u, B_d, C; matrices - projecting matrices(12) and(13); constants $\gamma \geq \gamma^*(5)$, $tol(\approx 0.001)$, $n = 0$.

(1) *Solve the LQR problem for given* $(A,\ B_u,\ Q,\ R)$ *and obtain SSF-gain* K_0 *and ARE-solution* P_0.

(2) *Define the matrix* $\tilde{A}_0 = A - BK_0$.

(3) *n-th iteration: solve ARE for* P_n:

$$P_n(\tilde{A}_n) + (\tilde{A}_n^{-T})P + Q + K_n^T R K_n + \frac{1}{\gamma^2}P_n B_d B_d^T P_n = 0$$

(4) *Update* $K_n : K_{n+1} = R^{-1}(B_u^T P_n + L_n)f$

(5) *Update* L_n: $L_{n+1} = RK_n - B^T P_n$

(6) *Update* $\tilde{A}_n : \tilde{A}_{n+1} = \tilde{A}_n - BK_{n+1}$

(7) *Check convergence: if* $\|P_{n+1} - P_n\| < tol$, *go to step 8; otherwise go to step 2 and set*

(8) $n = n + 1$.

(9) *End. Set* $K_f = K_{n+1}$ *and compute SOF gain* $K = K_f \cdot C^+$ *based on the SSF-gain* K_f.

(10) *Compute* $\|H_{zd}(j\omega)\|_\infty$ *and* $\|H_{zd}(j\omega)\|_2$ *for system* (1), (3) *with SOF-gain* K, *obtained at stage 8.*

Now we will apply this algorithm for the solution of the (1-5) problem to find SOF-gains for control of the longitudinal and lateral quadrotor motions and to apply these matrices for the design of the quadrotor path following the control system.

4. Case study

In this item, we will use the linearized mathematical model (1, 2) of the real quadrotor developed at the National Aviation University (Kiev). It has the following parameters: the total mass of quadrotor is m_Q= 5.5 kg, mass of single electromotor Foxtech X5010 KV288 m_{em}= 0.213 kg, length of the motor's arm l =0.343 m, the mass of the battery and payload m_{bp}= 4.65 kg, the maximal value of the rotation rate of each electromotor $\Omega_{\max} = 1809 rad/\sec$. This model was developed based on quadrotor design technique proposed in [2, 11, 13-15]. Numerical values of matrices $A,\ B_u,\ B_d,\ C$ appearing in (1) for control of quadrotor motion along Y-axis look as follows:

$$A=\begin{bmatrix} 0 & 1 & 0 & 0 & 0\\ 0 & -0.28 & 9.81 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 13.51\\ 0 & 0 & 0 & 0 & -8.33 \end{bmatrix},\ B_u=\begin{bmatrix} 0\\ 0\\ 0\\ 0\\ 10.42 \end{bmatrix},$$

$$B_d=\begin{bmatrix} 0\\ 1\\ 0.0285\\ 0\\ 0 \end{bmatrix},\ C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \tag{14}$$

Model (1), (14) for control of quadrotor motion along X-axis is the same as for Y-axis and the difference consists of the inverse sign of the a_{23} entry of matrix A: $a_{23}=-9.81$, [5, 9, 10]. That is why we will describe the control design procedure only for Y-axis.

We will begin with a synthesis of control law for the inner loop to stabilize the module of quadrotor linear velocity at the given level. In this case, state χ and output υ vectors (2) will have the following form:

$$\chi=[\chi_1,\chi_2,\chi_3,\chi_4,\chi_5]^T=[y,dy/dt,\phi,p,\Delta\Omega]^T,\ \ \upsilon=[y,dy/dt,\phi,p]^T. \tag{15}$$

and matrices $A,\ B_u,\ B_d,\ C$ will look like:

$$A_V=\begin{bmatrix} -0.28 & 9.81 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 13.51\\ 0 & 0 & 0 & -8.33 \end{bmatrix},B_{uV}=\begin{bmatrix} 0\\ 0\\ 0\\ 10.42 \end{bmatrix},$$

$$B_{dV}=\begin{bmatrix} 0\\ 1\\ 0.0285\\ 0 \end{bmatrix},\ \ C_V=\begin{bmatrix} 1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{16}$$

Choosing the weighting matrix Q_V, and scalar values R_V,γ^*,tol the algorithm of synthesis [6] described above as follows:

$$Q_V=diag([3\cdot 10^7,700,7\cdot 10^6,0]),R_V=3.7\cdot 10^4,\gamma^*=0.5,tol=0.001,$$

and applying this algorithm for the outer loop, we will obtain the following results: SOF-gain matrix $K_V=[25.85,92.14,16.61]$, eigenvalues of state propagation matrix of the closed-loop system $Eig=(-1.36\pm j48.01,-2.94\pm j2.89)$, minimal value of L_2 -gain in (5) $\lambda^*=0.37$.

Now we use the gain matrix $K_V$for closing the inner loop to obtain the resulting system with an inner closed-loop contour. Further, we apply the algorithm developed in [6] to the system under consideration to perform the second stage of the entire synthesis procedure, i.e. finding outer feedback for position control of quadrotor. Choosing weighting matrix Q, and scalar values R,γ^*,tol for the 2nd stage of the algorithm as follows:

$$Q=diag\left(\left[3\cdot 10^5,\quad 2.5\cdot 10^4,\ \ 7.5\cdot 10^4,\ \ 2.5\cdot 10^5\right]\right),R=10^3,\gamma^*=0.5,tol=0.001,$$

and applying this algorithm for the outer loop, we will obtain the following results: SOF-gain matrix $K_Y = [17.32, 6.67, 33.13, 7.87]$, eigenvalues of state propagation matrix of the closed-loop system $Eig = (-1.59 \pm j58.43, -2.415 \pm j2.42, -0.599)$, minimal value of L_2-gain in (5) $\lambda^* = 0.37$. Fig.2 shows simulation results of quadrotor motion control along the Y-axis.

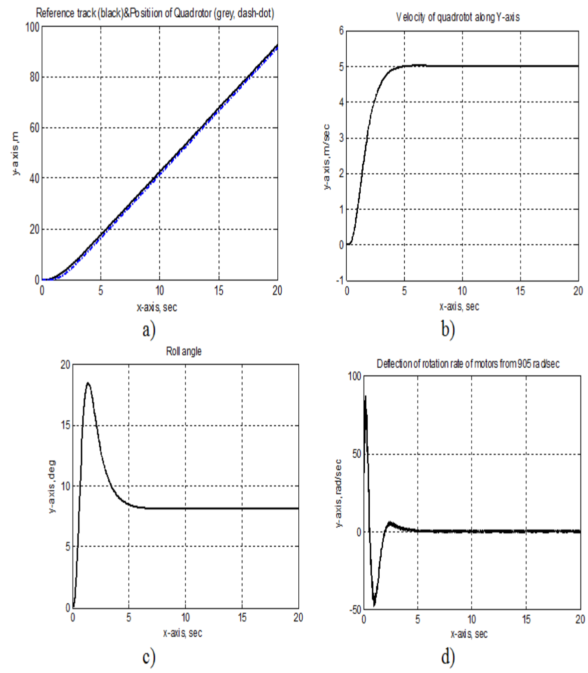

Figure 2. Results of simulation of motion control along the Y-axis: a) following command signal (black line) by quadrotor (grey, dash-dot line), b) transient process of velocity, c) transient process of roll angle, d) transient process of rotation rate increment of electromotor.

As it was mentioned above, results of synthesis of control law for motion control along the X-axis differ from the previous case only by signs of entries to the SOF-gain matrices:

$$K_V = [-25.85, 92.14, 16.61]; K_X = [-17.32, -6.67, 33.13, 7.87].$$

At this stage, it is possible to combine these motion control systems for the quadrotor path following control in the horizontal plane, assuming that it flies at a constant altitude.

First of all, we have mentioned that the procedure of synthesis of control laws for each axis based on the linearized models. Meanwhile, the simulation of path-following control uses all nonlinearities, which are immanent to the real system. These include the nonlinear interconnections between motions along the X-axis and Y-axis and saturation of the electro motors electronic speed control. The nonlinear interconnections appear in the initial mathematical model as follows [5, 9, 10, 16]:

$$\frac{d^2x}{dt^2} = -[\frac{\mu}{m_Q}\frac{dx}{dt} + g\frac{\tan\theta}{\cos\phi}] \cdot \cos\psi, \quad \frac{d^2y}{dt^2} = -[\frac{\mu}{m_Q}\frac{dy}{dt} - g\tan\phi] \cdot \sin\psi, \tag{17}$$

where μ is the coefficient defining the drag force [11],m_Q is the total mass of the quadrotor, and ψ is the heading angle of the quadrotor. Fig.3 presents a block diagram of the path following guidance system (GS) of the quadrotor in the horizontal plane.

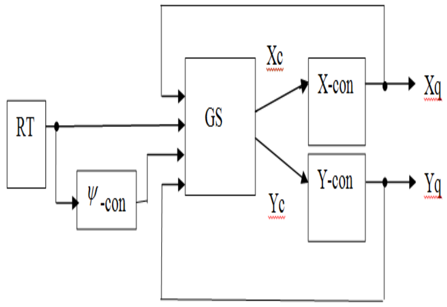

Figure 3. Block diagram of path following system in the horizontal plane.

In Fig.3 RT block represents the reference track generator producing command signals to GS and the heading control system $\psi - con$. GS determines the errors between the reference track and the actual position of the quadrotor and produces command signals X_C, Y_C to motion control systems $X - con$ and $Y - con$. The mathematical model of the heading control system for the aforementioned quadrotor has the following form [2, 11, 13-15]:

$$\chi = [\psi, r, \Delta\Omega]^T, \upsilon = [\psi, r]^T,$$

$$\frac{d\chi}{dt} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 8.064 \\ 0 & 0 & -8.33 \end{bmatrix} \chi + \begin{bmatrix} 0 \\ 0 \\ 0.57 \end{bmatrix} u_\psi, \upsilon = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \chi, \tag{18}$$

$$u_\psi = -K_\psi \upsilon,$$

where static output feedback is: $K_\psi = [k_\psi, k_r] = [20, 15]$, and r is the heading rate.
In GS we used the guidance control law for planar circular UAV motion, which was proposed in [12] and based on UAV kinematics and geometrical properties trajectory. It has the following form:

$$U_G = k_e e_{tr} + k_{\dot{e}} \dot{e}_{tr} + k_0, \tag{19}$$

where $U_G$is the control output of GS, $e_{tr}, \dot{e}_{tr}$ stands for cross-track error and error rate. The radius$R_C = 160m$ of RT-circle, UAV centripetal acceleration and velocity, and pseudo-target lead for line-of-sight from UAV to the reference track define gains $k_e, k_{\dot{e}}$ [12]. The term k_0 is defined by the centripetal acceleration of the quadrotor during circular motion [12] and it is equal to $k_0 = k_w \frac{V_{com}^2}{R_C g}$, where V_{com} is the command velocity; g is the gravity acceleration, and k_w is a weighting coefficient. In our case the gains are equal to: $k_e = 0.1$,$k_{\dot{e}} = 0.5$, and the constant term equals $k_0 = 0.005$. For the case of quadcopter control action (19) must be projected on X and Y axes:

$$X_C = U_G \cos\psi, Y_C = U_G \sin\psi. \tag{20}$$

Control actions (20) determine the direction of the velocity vector of the quadrotor required for following the circular path. Fig.4 represents simulation results of the quadrotor path following in the condition of a calm atmosphere.

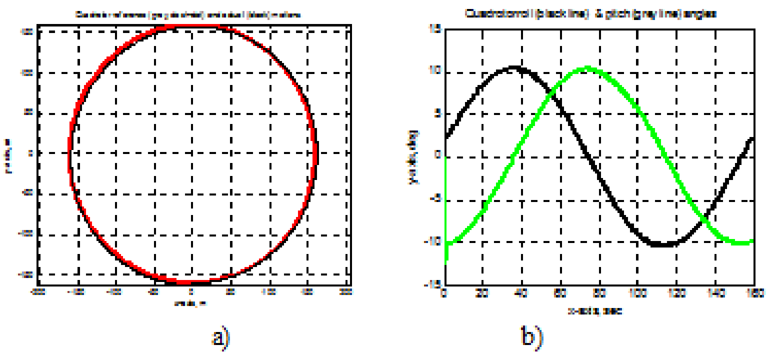

Figure 4. Simulation of the quadrotor path following in the calm atmosphere: a) quadrotor path (grey, dash-dot line) and reference track (black, solid line), b) roll angle (black line) and pitch angle (grey line) in *deg*.

Fig. 5 represents results of simulation of quadrotor path following, using Dryden model of the moderately disturbed atmosphere at low altitude [3]. As it was shown from Figures 4 and 5, proposed control laws provide acceptable accuracy of path following (2.2% in the calm atmosphere and practically the same in the disturbed atmosphere), effective suppressing of the atmospheric disturbance and acceptable values of the pitch and roll deflections (10o in the calm atmosphere and 20o in the disturbed atmosphere).

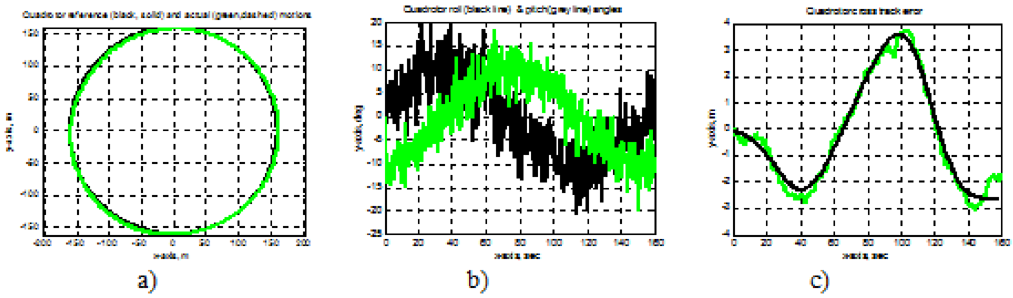

Figure 5. Simulation of the quadrotor path following in the disturbed atmosphere: a) quadrotor path (grey, dashed line) and reference track (black, solid line) in m, b) roll angle (black line) and pitch angle (grey line) in , c) cross-track error in the calm atmosphere (in m, black line) and the same error in the disturbed atmosphere (in m, grey line).

5. Conclusions

1. To simplify the solution of the quadrotor path planning and the path following problems; it is expedient in many practical cases to design flight control systems, considering quadrotor as a nonholonomic system. In this case, it is necessary to synthesize control laws for the stabilization of the module of the velocity vector and for control of its direction.
2. Following the peculiarities of quadrotor flight control, it is acceptable to design control laws for three spatial axes separately. Therefore, for planar motion at the constant altitude, it is possible to synthesize identical control systems for motion control along the X- and Y-axes.
3. For the sake of control laws' simplicity, it is desirable to choose them as the static output feedback (SOF) gains. The most effective approach for the synthesis of SOF-gains is the $H_\infty-$ parameterization of all stabilizing controllers, based on the iterative solutions of Riccati algebraic equations. This approach is utilized in the two-stage procedure for control laws design, where the inner contour constitutes the first stage of the design for velocity stabilization, meanwhile,

the second stage implies the position control.
4. Simulation of quadrotor flight in calm and disturbed atmosphere illustrated the efficiency of proposed control algorithms.

References

[1] Balandin, D.V., Kogan, M.M., (2010), Generalized optimal control as a trade-off between the optimal and optimal controls, Autom. Remote Control, 71(6), pp.993-1010.

[2] Beard, R., (2008), Quadrotor Dynamics and Control Rev 0.1, Available at: https://scholarsarchive.byu.edu/facpub/13252.

[3] Beard, R.W., McLain, T.W., (2012), Small Unmanned Aircraft. Theory and Practice, Princeton University Press, 300p.

[4] Boyd, S., El Ghaoui, L., Feron, E., Balakrishnan, V., (1994), Linear Matrix Inequalities in System and Control Theory, Philadelphia, PA SIAM, 416p.

[5] Castillo, P., Lozano, R., Dzul, A., (2005), Stabilization of a mini rotorcraft with four rotors, IEEE Control Systems Magazine, December, pp.45-55.

[6] Gadewadikar, J., Lewis, F., Xie, L., Kucera, V., Abu-Khalaf M., (2007), Parameterization of all stabilizing H_∞ static state-feedback gains: application to Output-feedback design, Automatica, 43, pp.1597-1604.

[7] Hoffman, G.M., Waslander, S.L., Tomlin, C.J., (2008), Quadrotor helicopter trajectory tracking control, AIAA Guidance, Navigation and Control Conference and Exhibition, 18-21 August Hawaii, Honolulu, pp.1-14.

[8] Komnatska, M.M., Tunik, A.A., (2009), Robust stabilization and optimization of flight control system with state feedback and fuzzy logics, NAU Proceedings, 4, pp.33-41.

[9] Larin, V.B, Tunik, A.A., (2018), Synthesis of the quad-rotor control algorithms in the basic flight modes, TWMS J. Pure Appl. Math,, 9(2), pp.147-158.

[10] Larin, V.B, Tunik, A.A., (2017), On problem of synthesis of control system for quadrocopter, Int. Appl. Mech., 53(3), pp.342-348.

[11] Leishman, R.C., Macdonald, J.C., Beard R.W., McLain T.W., (2014), Quadrotors and accelerometers, IEEE Control Systems Magazine, February, pp.28-41.

[12] Park, S., Deyst, J., How, J.P., (2004), A new nonlinear guidance logic for trajectory tracking, in Proc. of 1he AIAA Guidance, Navigation and Control Conf., Providence, RI, USA, August pp.941-956.

[13] Phang, S.K., Li, K., Chen, B.M., Lee, T.H., (2015), Systematic Design Methodology and Construction of Micro Aerial Quadrotor Vehicles, In book: Handbook of Unmanned Aerial Vehicles, Kimon P. Valavanis and George J. Vachtsevanos Editors, Springer Science+Business Media Dordrecht, pp.182-206.

[14] Phang, S.K., Cai, C., Chen, B.M., Lee, T.H., (2012), Design and mathematical modeling of a 4-standard-propeller (4SP) quadrotor, Proceedings of the 10th World Congress on Intelligent Control and Automation, Beijing, China, pp.3270-3275.

[15] Tsay, T.S., (2014), Guidance and control laws for quadrotor UAV, WSEAS Trans. Syst. Control, 9, pp.606-613.

[16] Tunik, A.A., (2018), Simplified path tracking control laws for quadrotor considered as nonholonomic system, MSNMC, pp.83-90.

[17] Tunik, A.A., Komnatska, M.M., Klipa, A.M., (2010), LMI-based static output feedback design for rotating solid body, 1st International Conference Methods and Systems of Navigation and Motion Control, Proceedings, NAU, Kiev, pp.10-14.

Vladimir B.Larin, for a photograph and biography, see TWMS J. Pure Appl. Math., V.2, N.1, 2011, p.160.

Anatoly A.Tunik, for a photograph and biography, see TWMS J. Pure Appl. Math., V.7, N.2, 2016, p.166.