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A REMARK ABOUT ENERGY DECAY OF A THERMOELASTICITY PROBLEM

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Abstract. Under some restrictions on the parameters of the system we prove that solutions of the initial boundary value problem for the one dimensional porous - thermo - elasticity system of equations under consideration tend to zero as $t \to \infty$ with an exponential rate.

Keywords: thermo-elasticity, exponential decay, energy method.

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1. Introduction

We study the following initial boundary value problem:

$$\rho u_{tt} = \mu u_{xx} + b\phi_x - \beta\theta_x, x \in (0,\pi), t > 0, \tag{1}$$

$$J\phi_{tt} = \delta\phi_{xx} - bu_x - \xi\phi + m\theta - \tau\phi_t, x \in (0,\pi), t > 0, \tag{2}$$

$$c\theta_t = k\theta_{xx} - \beta u_{xt} - m\phi_t, x \in (0,\pi), t > 0, \tag{3}$$

$$u(0,t) = u(\pi,t) = \phi_x(0,t) = \phi_x(\pi,t) = \theta(0,t) = \theta(\pi,t) = 0, t > 0, \tag{4}$$

$$\begin{aligned} &u(x,0) = u_0(x), u_t(x,0) = u_1(x), \\ &\phi(x,0) = \phi_0(x), \phi_t(x,0) = \phi_1(x), \theta(x,0) = \theta_0(x), x \in (0,\pi). \end{aligned} \tag{5}$$

Here $\rho, \mu, b, \beta, J, \delta, \xi, m, \tau, c, k$ are given positive numbers, $u_0(x), u_1(x), \phi_0(x), \phi_1(x)$, and $\theta_0(x)$ are given initial functions, $u(x,t)$, $\phi(x,t)$ and $\theta(x,t)$ are unknown functions that represent the displacement of the solid material, the volume fraction and the temperature, respectively. The problem of exponential decay of solutions to the system of equations (1)-(3) was considered in the following papers:

- Cassas and Quintanilla [1] established exponential stability of solutions to the initial boundary value problem for the system of equations (1)-(3) under the boundary conditions

$$u(0,t) = u(\pi,t) = \phi_x(0,t) = \phi_x(\pi,t) = \theta_x(0,t) = \theta_x(\pi,t) = 0, t > 0 \tag{6}$$

  by using the semigroup approach of Liu and Zheng [2].
- By using the energy method Rivera and Quintanilla [4] proved exponential stability of solutions to the system (1)-(3) under the boundary conditions (6) when $\tau = 0, b^2 < \xi\mu$ and $m(\beta b - m\mu) > 0$.

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- Soufyane, Afilal, Aouam and Chacha [5] obtained a result about exponential decay of solutions of the system (1)-(3) with $\tau = 0$ under the boundary conditions

$$u(0,t) = \phi(0,t) = \theta(0,t) = \theta(\pi, T) = 0,$$

$$u(\pi,t) = -\int_0^t g_1(t-s)[\mu u_x(\pi,s) + b\phi(\pi,s)]ds,$$

$$\phi(\pi,t) = -\int_0^t g_2(t-s)\alpha\phi_x(\pi,s)ds.$$

In what follows we will use the following lemma established in [3]:

Lemma 1.1. *Let v be the solution of the inhomogeneous scalar wave equation*

$$v_{tt} - v_{xx} = f(x,t),\ 0 < x < \pi,\ 0 < t < T, \tag{7}$$

$$v(x,0) = v_0(x),\ v_t(x,0) = v_1(x), v(0,t) = v(\pi,t) = 0, \tag{8}$$

where $\Omega = (0,\pi)$. The functions v_0, v_1 and f belong to $H_0^1(\Omega) \cap H^2(\Omega)$, $H_0^1(\Omega)$ and $H^1(0,T;L^2(\Omega))$ respectively. Then, the identity

$$\frac{1}{4}\pi[v_x^2(\pi,t) + v_x^2(0,t)] = \frac{d}{dt}((x - \frac{\pi}{2})v_t, v_x) + \frac{1}{2}[||v_x^2|| + ||v_t^2||] - ((x - \frac{\pi}{2})f, v_x) \tag{9}$$

holds.

2. Energy decay of solutions

Theorem 2.1. *Let $\{u, \phi, \theta\}$ be a solution of the problem (1)-(5), $2b^2 < \mu\xi$ and $b\beta > 8\mu m$,then there exists positive constant λ such that*

$$E(t) \le Cexp(-\lambda t)E(0),$$

Proof. First we derive the energy equality for solutions of the problem. Multiplying in $L^2(0,\pi)$ the equation (1) by u_t, the equation (2) by ϕ_t, the equation (3) by θ and adding the obtained relations we get:

$$\frac{d}{dt}E_1(t) = -\tau||\phi_t||^2 - k||\theta_x||^2, \tag{10}$$

where

$$E_1(t) = \frac{1}{2}\left[\rho||u_t||^2 + \mu||u_x||^2 + J||\phi_t||^2 + \delta||\phi_x||^2 + \xi||\phi||^2 + c||\theta||^2 + 2b(\phi, u_x)\right]. \tag{11}$$

Let us differentiate the equations (1)-(3) with respect to t, multiply in $L^2(0,\pi)$ the obtained equations by u_{tt}, ϕ_{tt}, θ_t respectively and sum the obtained equalities

$$\frac{d}{dt}E_2(t) = -\tau\|\phi_{tt}\|^2 - k\|\theta_{xt}\|^2. \tag{12}$$

Here

$$E_2(t) = \frac{1}{2}[\rho||u_{tt}||^2 + \mu||u_{xt}||^2 + J||\phi_{tt}||^2 + \delta||\phi_{xt}||^2 + \xi||\phi_t||^2 + c||\theta_t||^2 + 2b(\phi_t, u_{xt})]. \tag{13}$$

Multiplication in $L^2(0,\pi)$ of (1) by $-u_{xxt}$, (2) by $-\phi_{xxt}$ and (3) by $-\theta_{xx}$ gives

$$\frac{d}{dt}E_3(t) = -\tau||\phi_{xt}||^2 - k||\theta_{xx}||^2 + \\ + \beta[\theta_x(\pi,t)u_{xt}(\pi,t) - \theta_x(0,t)u_{xt}(0,t)] + m[\theta_x(\pi,t)\phi_t(\pi,t) - \theta_x(0,t)\phi_t(0,t)],$$

where

$$E_3(t) = \frac{1}{2}[\rho||u_{xt}||^2 + \mu||u_{xx}||^2 + J||\phi_{xt}||^2 + \delta||\phi_{xx}||^2 + \xi||\phi_x||^2 + c||\theta_x||^2 + 2b(\phi_x, u_{xx})]. \quad (14)$$

Employing the Lemma 1.1 with $v = u_t$ and $f = b\phi_{xt} - \beta\theta_{xt}$, we obtain

$$\frac{d}{dt}\rho(u_{tt},(x-\frac{\pi}{2})u_{xt}) = -\frac{\rho}{2}||u_{tt}||^2 - \frac{\mu}{2}||u_{xt}||^2 + \\ + \frac{\mu\pi}{4}(u_{xt}^2(\pi,t) + u_{xt}^2(0,t)) + b((x-\frac{\pi}{2})u_{xt},\phi_{xt}) - \beta(\theta_{xt},(x-\frac{\pi}{2})u_{xt}), \quad (15)$$

where

$$H(t) = -\rho(u_{tt},(x-\frac{\pi}{2})u_{xt}). \quad (16)$$

Next we multiply the equation (1) by $-u_{xx}$ and integrate over $(0,\pi)$

$$-(\rho u_{tt}, u_{xx}) = -\mu||u_{xx}||^2 - b(\phi_x, u_{xx}) + \beta(\theta_x, u_{xx}).$$

Thus,

$$\frac{d}{dt}K(t) = \rho||u_{xt}||^2 - \mu||u_{xx}||^2 - b(\phi_x, u_{xx}) + \beta(\theta_x, u_{xx}), \quad (17)$$

where

$$K(t) = \rho(u_x, u_{xt}). \quad (18)$$

Multiply (3) by u_{xt} and integrating over $(0,\pi)$, we get

$$(c\theta_t + m\phi_t, u_{xt}) = (k\theta_{xx}, u_{xt}) - \beta||u_{xt}||^2.$$

So

$$\frac{d}{dt}P(t) = -(c\theta_x + m\phi_x, u_{tt}) - \beta||u_{xt}||^2 + k(\theta_{xx}, u_{xt}), \quad (19)$$

where

$$P(t) = (c\theta + m\phi, u_{xt}). \quad (20)$$

Multiplying (2) by ϕ and integrating over $(0,\pi)$we obtain

$$J(\phi_{tt},\phi) = -\delta||\phi_x||^2 - b(u_x,\phi) - \xi||\phi||^2 + m(\theta,\phi) - \tau(\phi_t,\phi).$$

So

$$\frac{d}{dt}I(t) = J||\phi_t||^2 - \delta||\phi_x||^2 - b(u_x,\phi) - \xi||\phi||^2 + m(\theta,\phi), \quad (21)$$

where

$$I(t) = J(\phi,\phi_t) + \frac{\tau}{2}||\phi||^2. \quad (22)$$

Multiplying (1) by u and integrating over $(0,\pi)$ we obtain

$$\rho(u_{tt},u) = -\mu||u_x||^2 - b(\phi,u_x) + \beta(\theta,u_x)$$

So

$$\frac{d}{dt}L(t) = \rho||u_t||^2 - \mu||u_x||^2 - b(\phi,u_x) + \beta(\theta,u_x), \tag{23}$$

where

$$L(t) = \rho(u_t,u). \tag{24}$$

Let us consider the function

$$\begin{aligned} E(t) := \gamma E_1(t) + \nu E_2(t) + \eta E_3(t) + \epsilon_0 I(t) + \epsilon_1 P(t) + \epsilon_2 H(t) + \epsilon_3 K(t) + \epsilon_4 L(t) = \\ = \frac{\gamma}{2}[[\rho||u_t||^2 + \mu||u_x||^2 + J||\phi_t||^2 + \delta||\phi_x||^2 + \xi||\phi||^2 + c||\theta||^2 + 2b(\phi,u_x)] + \\ + \frac{\nu}{2}[\rho||u_{tt}||^2 + \mu||u_{xt}||^2 + J||\phi_{tt}||^2 + \delta||\phi_{xt}||^2 + \xi||\phi_t||^2 + c||\theta_t||^2 + 2b(\phi_t,u_{xt})] + \\ + \frac{\eta}{2}[\rho||u_{xt}||^2 + \mu||u_{xx}||^2 + J||\phi_{xt}||^2 + \delta||\phi_{xx}||^2 + \xi||\phi_x||^2 + c||\theta_x||^2 + 2b(\phi_x,u_{xx})]] + \\ + \epsilon_0[J(\phi,\phi_t) + \frac{\tau}{2}||\phi||^2] + \epsilon_1[(c\theta + m\phi, u_{xt})] + \epsilon_2[-\rho(u_{tt},(x-\frac{\pi}{2})u_{xt})] + \\ + \epsilon_3[\rho(u_x,u_{xt})] + \epsilon_4[\rho(u,u_t)]. \end{aligned} \tag{25}$$

It is not difficult to see that

$$\begin{aligned} \frac{d}{dt}E(t) = \gamma[-\tau||\phi_t||^2 - k||\theta_x||^2] - \nu[\tau\|\phi_{tt}\|^2 + k\|\theta_{xt}\|^2] + \eta[-\tau||\phi_{xt}||^2 - k||\theta_{xx}||^2 + \\ + \beta(\theta_x,u_{xt})|_0^\pi + m(\theta_x,\phi_t)|_0^\pi] + \epsilon_0[J||\phi_t||^2 - \delta||\phi_x||^2 - \underbrace{b(\phi,u_x)}_{e} - \xi||\phi||^2 + \underbrace{m(\theta,\phi)}_{a}] + \\ + \epsilon_1[\underbrace{k(\theta_{xx},u_{xt})}_{b} - \beta||u_{xt}||^2 - (c\theta_x + m\phi_x, u_{tt})] + \\ + \epsilon_2[\frac{\rho}{2}||u_{tt}||^2 + \frac{\mu}{2}||u_{xt}||^2 - \frac{\mu\pi}{4}(u_{xt}^2(\pi,t) + u_{xt}^2(0,t)) - b((x-\frac{\pi}{2})u_{xt},\phi_{xt}) + \beta(\theta_{xt},(x-\frac{\pi}{2})u_{xt})] + \\ + \epsilon_3[\rho||u_{xt}||^2 - \mu||u_{xx}||^2 - \underbrace{b(\phi_x,u_{xx})}_{c} + \underbrace{\beta(\theta_x,u_{xx})}_{d}] + \epsilon_4[\rho||u_t||^2 - \mu||u_x||^2 - b(\phi,u_x) + \beta(\theta,u_x)]. \end{aligned}$$

Due to Cauchy inequality and the Poincare inequality we get the estimates

$$a = m(\theta,\phi) \le \frac{\xi}{4}||\phi||^2 + \frac{m^2 d_0}{\xi}||\theta_x||^2, b = k(\theta_{xx},u_{xt}) \le \frac{\beta}{2}||u_{xt}||^2 + \frac{k^2}{2\beta}||\theta_{xx}||^2$$

$$c = b(\phi_x,u_{xx}) \le \frac{\mu}{4}||u_{xx}||^2 + \frac{b^2}{\mu}||\phi_x||^2, d = \beta(\theta_x,u_{xx}) \le \frac{\mu}{4}||u_{xx}||^2 + \frac{\beta^2}{\mu}||\theta_x||^2$$

$$e = b(\phi,u_x) \le \frac{b^2}{\xi}||u_x||^2 + \frac{\xi}{4}||\phi||^2.$$

Thus

$$\frac{d}{dt}E(t) \le \gamma[-\tau||\phi_t||^2 - k||\theta_x||^2] - \nu[\tau\|\phi_{tt}\|^2 + k\|\theta_{xt}\|^2] + \eta[-\tau||\phi_{xt}||^2 - k||\theta_{xx}||^2+$$
$$+\beta(\theta_x, u_{xt})|_0^\pi + m(\theta_x,\phi_t)|_0^\pi] + \epsilon_0[J||\phi_t||^2 - \delta||\phi_x||^2 - \frac{\xi}{2}||\phi||^2 + \frac{b^2}{\xi}||u_x||^2 + \frac{m^2 d_0}{\xi}||\theta_x||^2]+$$
$$+\epsilon_1[\frac{k^2}{2\beta}||\theta_{xx}||^2 - \frac{\beta}{2}||u_{xt}||^2 - (c+\frac{\beta m}{b})(\theta_x,u_{tt}) + \frac{\mu m}{b}(u_{xx},u_{tt}) - \frac{\rho m}{b}||u_{tt}||^2]+$$
$$+\epsilon_2[\frac{\rho}{2}||u_{tt}||^2 + \frac{\mu}{2}||u_{xt}||^2 - \frac{\mu\pi}{4}(u_{xt}^2(\pi,t)+u_{xt}^2(0,t)) - b((x-\frac{\pi}{2})u_{xt},\phi_{xt}) + \beta(\theta_{xt},(x-\frac{\pi}{2})u_{xt})]+$$
$$+\epsilon_3[\rho||u_{xt}||^2 - \frac{\mu}{2}||u_{xx}||^2 + \frac{b^2}{\mu}||\phi_x||^2 + \frac{\beta^2}{\mu}||\theta_x||^2] + \epsilon_4[\rho||u_t||^2 - \mu||u_x||^2 - b(\phi,u_x) + \beta(\theta,u_x)].$$

Here we have used the equality

$$-(c\theta_x + m\phi_x, u_{tt}) = -(c+\frac{\beta m}{b})(\theta_x,u_{tt}) + \frac{\mu m}{b}(u_{xx},u_{tt}) - \frac{\rho m}{b}||u_{tt}||^2$$

that follows from (1). So we have

$$\frac{d}{dt}E(t) \le \gamma[-\tau||\phi_t||^2 - k||\theta_x||^2] - \nu[\tau\|\phi_{tt}\|^2 + k\|\theta_{xt}\|^2] + \eta[-\tau||\phi_{xt}||^2 - k||\theta_{xx}||^2+$$
$$+\underbrace{\beta(\theta_x, u_{xt})|_0^\pi}_{l} + \underbrace{m(\theta_x,\phi_t)}_{m}|_0^\pi] + \epsilon_0[J||\phi_t||^2 - \delta||\phi_x||^2 - \frac{\xi}{2}||\phi||^2 + \frac{b^2}{\xi}||u_x||^2 + \frac{m^2 d_0}{\xi}||\theta_x||^2]+$$
$$+\epsilon_1[\frac{k^2}{2\beta}||\theta_{xx}||^2 - \frac{\beta}{2}||u_{xt}||^2 - \underbrace{(c+\frac{\beta m}{b})(\theta_x,u_{tt})}_{f} + \underbrace{\frac{\mu m}{b}(u_{xx},u_{tt})}_{g} - \frac{\rho m}{b}||u_{tt}||^2]+$$
$$+\epsilon_2[\frac{\rho}{2}||u_{tt}||^2 + \frac{\mu}{2}||u_{xt}||^2 - \frac{\mu\pi}{4}(u_{xt}^2(\pi,t)+u_{xt}^2(0,t)) - \underbrace{b((x-\frac{\pi}{2})u_{xt},\phi_{xt})}_{h} + \underbrace{\beta(\theta_{xt},(x-\frac{\pi}{2})u_{xt})}_{i}]+$$
$$+\epsilon_3[\rho||u_{xt}||^2 - \mu||u_{xx}||^2 + \frac{b^2}{\mu}||\phi_x||^2 + \frac{\beta^2}{\mu}||\theta_x||^2] + \epsilon_4[d_0\rho||u_{xt}||^2 - \mu||u_x||^2 - \underbrace{b(\phi,u_x)}_{j} + \underbrace{\beta(\theta,u_x)}_{k}].$$

Let us estimate the terms $(f),(g),(h),(i),(j),(k),(l),(m)$ on the right hand side of the last inequality

$$f = (c+\frac{m\beta}{b})(\theta_x,u_{tt}) \le \frac{\rho m}{4b}||u_{tt}||^2 + \frac{b}{\rho m}(c+\frac{m\beta}{b})^2||\theta_x||^2,$$

$$g = \frac{\mu m}{b}(u_{xx},u_{tt}) \le \frac{\rho m}{4b}||u_{tt}||^2 + \frac{m\mu^2}{\rho b}||u_{xx}||^2,$$

$$h = b((x-\frac{\pi}{2})u_{xt},\phi_{xt}) \le \frac{\epsilon_1\beta}{8\epsilon_2}||u_{xt}||^2 + \frac{b^2\pi^2\epsilon_2}{2\epsilon_1\beta}||\phi_{xt}||^2,$$

$$i = \beta(\theta_{xt},(x-\frac{\pi}{2})u_{xt}) \le \frac{\epsilon_1\beta}{8\epsilon_2}||u_{xt}||^2 + \frac{\pi^2\epsilon_2\beta}{2\epsilon_1}||\theta_{xt}||^2,$$

$$j = b(\phi,u_x) \le \frac{\mu}{4}||u_x||^2 + \frac{b^2}{\mu}||\phi||^2,$$

$$k = \beta(\theta,u_x) \le \frac{\mu}{4}||u_x||^2 + \frac{\beta^2 d_0}{\mu}||\theta_x||^2,$$

$$l = \eta\beta(\theta_x,u_{xt})|_0^\pi \le \eta\beta(\frac{1}{2\epsilon}(\theta_x^2(\pi,t)+\theta_x^2(0,t)) + \frac{\epsilon}{2}(u_{xt}^2(\pi,t) + (u_{xt}^2(0,t))))$$

$$\leq \frac{2\eta^2\beta^2}{\mu\pi\epsilon_2}\sup_{x\in(0,\pi)}\theta_x^2+\frac{\epsilon_2\mu\pi}{4}(u_{xt}^2(\pi,t)+(u_{xt}^2(0,t)))\leq$$
$$\leq \frac{2\eta^2\beta^2}{\mu\pi\epsilon_2}(c_1||\theta_x||||\theta_x||_{H^1})+\frac{\epsilon_2\mu\pi}{4}(u_{xt}^2(\pi,t)+(u_{xt}^2(0,t)))\leq$$
$$\leq \frac{k}{4}||\theta_x||_{H^1}+\frac{4\eta^4\beta^4c_1^2}{k\mu^2\pi^2\epsilon_2^2}||\theta_x||^2+\frac{\epsilon_2\mu\pi}{4}(u_{xt}^2(\pi,t)+(u_{xt}^2(0,t))),$$

$$m=\eta m(\theta_x,\phi_t)|_0^\pi \leq \eta m(\frac{\epsilon}{2}(\theta_x^2(\pi,t)+\theta_x^2(0,t))+\frac{1}{2\epsilon}((\phi_t^2(\pi,t)+\phi_t^2(0,t)))\leq$$
$$\leq \eta m(\epsilon\sup_{x\in(0,\pi)}\theta_x^2+\frac{1}{\epsilon}\sup_{x\in(0,\pi)}\phi_t^2)\leq$$
$$\leq \frac{k}{2}\sup_{x\in(0,\pi)}\theta_x^2+\frac{2\eta^2m^2}{k}\sup_{x\in(0,\pi)}\phi_t^2\leq$$
$$\leq \frac{k}{2}c_1||\theta_x||||\theta_x||_{H^1}+\frac{2\eta^2m^2c_1}{k}||\phi_t||||\phi_t||_{H^1}\leq$$
$$\leq \frac{k}{4}||\theta_x||^2+\frac{kc_1^2}{4}||\theta_x||_{H^1}^2+\frac{\eta^2m^2}{k}(c_1^2\epsilon_*||\phi_t||^2+\frac{1}{2\epsilon_*}||\phi_t||_{H^1}^2)\leq$$
$$\leq \frac{k}{4}||\theta_x||^2+\frac{kc_1^2}{4}||\theta_x||_{H^1}^2+\frac{2\eta^4m^4c_1^2}{k^2\tau}||\phi_t||^2+\frac{\tau}{2}||\phi_t||_{H^1}^2\leq$$
$$\leq \frac{k}{4}(1+c_1^2)||\theta_x||^2+\frac{kc_1^2}{4}||\theta_{xx}||^2+(\frac{\tau}{2}+\frac{2\eta^4m^4c_1^2}{k^2\tau})||\phi_t||^2+\frac{\tau}{2}||\phi_{xt}||^2.$$

So we have,

$$\frac{d}{dt}E(t)\leq -[\gamma\tau-\frac{\tau}{2}-\frac{2\eta^4m^4c_1^2}{k^2\tau}-J\epsilon_0]||\phi_t||^2-$$
$$-[\gamma k-\frac{k}{4}(1+c_1^2)-\frac{k}{4}-\frac{4\eta^4\beta^4c_1^2}{k\mu^2\pi^2\epsilon_2^2}-\frac{m^2d_0}{\xi}\epsilon_0-\frac{b}{\rho m}(c+\frac{m\beta}{b})^2\epsilon_1-\frac{\beta^2}{\mu}\epsilon_3-\frac{\beta^2d_0}{\mu}\epsilon_4]||\theta_x||^2-$$
$$-[\nu k-\frac{\pi^2\epsilon_2^2\beta}{2\epsilon_1}]||\theta_{xt}||^2-[\eta\tau-\frac{\tau}{2}-\frac{b^2\pi^2\epsilon_2^2}{2\epsilon_1\beta}]||\phi_{xt}||^2-[\eta k-\frac{kc_1^2}{4}-\frac{k}{4}-\frac{k^2\epsilon_1}{2\beta}]||\theta_{xx}||^2-$$
$$-\nu\tau||\phi_{tt}||^2-[\delta\epsilon_0-\frac{b^2\epsilon_3}{\mu}]||\phi_x||^2-[\frac{\xi\epsilon_0}{2}-\frac{b^2\epsilon_4}{\mu}]||\phi||^2-[\frac{\mu\epsilon_4}{2}-\frac{b^2\epsilon_0}{\xi}]||u_x||^2-$$
$$-[\frac{\beta\epsilon_1}{2}-\frac{\mu\epsilon_2}{2}-\frac{\beta\epsilon_1}{4}-\rho\epsilon_3-\rho d_0\epsilon_4]||u_{xt}||^2-$$
$$-[\frac{\rho m\epsilon_1}{2b}-\frac{\rho\epsilon_2}{2}]||u_{tt}||^2-[\frac{\mu\epsilon_3}{2}-\frac{m\mu^2}{\rho b}\epsilon_1]||u_{xx}||^2. \quad (26)$$

We can choose $\epsilon_i, i=0,...,4$ so that

$$\gamma>\frac{1}{2}+\frac{2\eta^4m^4c_1^2}{k^2\tau^2}+\frac{J}{\tau}\epsilon_0,$$
$$\nu>\frac{1}{k}\frac{\pi^2\beta}{2}\frac{\epsilon_2^2}{\epsilon_1},$$
$$\eta>\frac{1}{2}+\frac{b^2\pi^2}{2\beta\tau}\frac{\epsilon_2^2}{\epsilon_1},$$
$$\eta>\frac{1}{4}+\frac{c_1^2}{4}+\frac{k}{2\beta}\epsilon_1,$$

$$\epsilon_0 > \frac{b^2}{\delta\mu}\epsilon_3,$$

$$\epsilon_0 > \frac{2b^2}{\mu\xi}\epsilon_4, \tag{27}$$

$$\epsilon_4 > \frac{2b^2}{\xi\mu}\epsilon_0, \tag{28}$$

$$\frac{\beta}{4}\epsilon_1 > \frac{\mu}{2}\epsilon_2 + \rho\epsilon_3 + \rho d_0\epsilon_4, \tag{29}$$

$$\epsilon_1 > \frac{b}{m}\epsilon_2,$$

$$\epsilon_3 > \frac{2m\mu}{\rho b}\epsilon_1, \tag{30}$$

$$k\gamma > \frac{k}{4}(1 + c_1^2) + \frac{k}{4} + \frac{4\eta^4\beta^4c_1^2}{k\mu^2\pi^2\epsilon_2^2} + \frac{m^2d_0}{\xi}\epsilon_0 + \frac{b}{\rho m}(c + \frac{m\beta}{b})^2\epsilon_1 + \frac{\beta^2}{\mu}\epsilon_3 + \frac{\beta^2d_0}{\mu}\epsilon_4.$$

It is easy to see that (27) and (28) are satisfied if

$$2b^2 < \xi\mu$$

and (29) and (30) are satisfied if

$$b\beta > 8\mu m.$$

So by using Cauchy inequality for (25) and from (26) we conclude that there exists a positive constant λ such that

$$\frac{d}{dt}E(t) \leq -\lambda E(t).$$

That means

$$E(t) \leq E(0)e^{-\lambda t}.$$

So we have proved that the energy decays exponentially. □

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