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NUMERICAL SOLUTION OF THE SYSTEM OF LINEAR FREDHOLM INTEGRAL EQUATIONS BASED ON DEGENERATING KERNELS

S. KARIMI[1], M. JOZI[1]

Abstract. In this paper, a new numerical method is proposed for solving system of linear Fredholm integral equations of the second kind. This method is based on Taylor-series expansion which degenerates kernels and reduces the system of integral equations to a block algebraic linear system. Convergence analysis is investigated and test problems are given to illustrate the robustness and efficiency of the new method.

Keywords: systems of Fredholm integral equations, degenerate kernel, Taylor-series expansion, block algebraic system, Banach space.

AMS Subject Classification: 45A05, 45B05, 45F05.

1. Introduction

We consider the system of linear Fredholm integral equations of the form:

$$\mathcal{U}(x) = \mathcal{F}(x) + \lambda \int_\Gamma \mathcal{K}(x,y)\mathcal{U}(y)dy, \ x \in \Gamma = [0,1], \tag{1}$$

with

$$\begin{aligned} \mathcal{U}(x) &= [u_1(x), u_2(x), ..., u_s(x)]^T, \\ \mathcal{F}(x) &= [f_1(x), f_2(x), ..., f_s(x)]^T, \\ \mathcal{K}(x,y) &= [K_{ij}(x,y)] \ i,j = 1,2,...,s, \end{aligned}$$

where the kernel $\mathcal{K}$ is continuous, $\mathcal{F}$ is given and $\mathcal{U}$ is the solution to be determined. Assuming that $\mathcal{X}$ is a Banach space, we can also write the equation (1) as follows

$$(I - \lambda\mathbb{K})\mathcal{U} = \mathcal{F}, \tag{2}$$

where

$$\begin{aligned} \mathbb{K} &: \ \mathcal{X} \to \mathcal{X} \\ \mathbb{K}\mathcal{U} &= \int_\Gamma \mathcal{K}(x,y)\mathcal{U}(y)dy. \end{aligned}$$

Recently, many different methods have been proposed to approximate the solution of integral equations systems [2, 5, 6]. Babolian et al. used Adomian decomposition method to obtain the solution of system (1) [3]. The Homotopy perturbation method [7] and its modification [8] were proposed by javidi and Golbabai. Maleknejad et al. presented a Taylor expansion method for a second kind Fredholm integral equation system with smooth or weakly singular kernel [14]. Their proposed method reduce the system of integral equation to linear system of ordinary differential equations. Golbabai and Keramati presented a simple method to approximate the solution

[1] Department of Mathematics, Faculty of Sciences, Persian Gulf University, Bushehr, Iran
e-mail: karimi@pgu.ac.ir, mjoozi@mehr.pgu.ac.ir
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of system of linear integral equations of the second kind based on Adomian's decomposition method [9]. Convergence analysis of Sinc-collocation method for approximating the solution of the integral equations system was proposed by Rashidinia and Zarebnia [16]. Triangular functions method for the solution of Fredholm integral equations system has been proposed by Almasieh and Roodaki in [1]. Moreover, Jafarian and Measoomynia used Feed-back neurol networks(NNs) approach for finding the approximate solution of (1) [10]. They substituted the Nth truncation of the Taylor expansion for unknown function in the origin system. It is well known that the Taylor expansion is a powerful tool to obtain the solution of different problems in numerical analysis. In recent years, many different methods based on Taylor expansion have been proposed in integral equation problems [10, 12, 13, 14, 15]. Since the Fredholm integral equation with degenerate kernel, can be solved easily [11], in this paper, starting from systems of equations having smooth kernels, we reduce to solve systems of equations having degenerate kernels. To do this we expand each kernel $k_{i,j}, i, j = 1, \dots, s$, with respect to the variable y in a suitable point c belonging to Γ. The approximate solution of system (1) can be obtained by solving the resulting block algebraic system of equation.

The organization of the paper is as follows. In Section 2, we establish the new method for solving the Fredholm integral system of the second kind. We will discuss convergence analysis of this method in Section 3. The numerical results are given in Section 4 to show the effectiveness of the proposed method. Finally, we make some concluding remarks in Section (5).

2. Description of the proposed method

First, consider the following Fredholm integral equation

$$u(x) = f(x) + \lambda \int_\Gamma k(x,y)u(y)dy,\ x \in \Gamma. \tag{3}$$

Suppose that kernel k and its derivatives of any order with respect to variable y exist in a neighborhood of a point $c \in \Gamma$. In this case $k(x, y)$ can be approximated by using the $N - th$ truncation of its Taylor-series expansion with respect to y at the point (x, c):

$$k(x,y) \simeq k(x,c) + (y-c)\frac{\partial k}{\partial y}|_{(x,c)} + \frac{(y-c)^2}{2!}\frac{\partial^2 k}{\partial y^2}|_{(x,c)} + ... + \frac{(y-c)^{N-1}}{(N-1)!}\frac{\partial^{N-1} k}{\partial y^{N-1}}|_{(x,c)} \tag{4}$$

which obtained by using Nth truncation of the Taylor-series expansion. Substituting the equation (4) into (3), we get the following degenerate integral equation

$$u_N(x) = f(x) + \lambda \sum_{i=1}^{N} a_i(x) \int_\Gamma b_i(y)u_N(y)dy, \tag{5}$$

where

$$a_i(x) = \frac{1}{(i-1)!}\frac{\partial^{i-1} k}{\partial y^{i-1}}|_{(x,c)},\ b_i(y) = (y-c)^{i-1},\ i = 1, 2, ..., N.$$

Following [11] we know that solution of integral equations (5) leads to a system of N linear algebraic equations that is easily solvable.

Now we want to generalize the above method for solving the system of integral equations (1). For convenience, we consider rth equation of system (1) as

$$u_r(x) = f_r(x) + \lambda \sum_{p=1}^{s} \int_\Gamma K_{rp}(x,y)u_p(y)dy,\ r = 1, 2, ..., s. \tag{6}$$

Suppose that all kernels K_{rp}, $r,p=1,2,...,s$ have the requirements listed for kernel of equation (3). Therefore, similar to (4) the kernels $K_{rp}(x,y)$ can be approximated as follows:

$$\begin{aligned} K_{rp}(x,y) &\simeq K_{rp}(x,c)+(y-c)\tfrac{\partial K_{rp}}{\partial y}|_{(x,c)}+\tfrac{(y-c)^2}{2!}\tfrac{\partial^2 K_{rp}}{\partial y^2}|_{(x,c)} \\ &+...+\tfrac{(y-c)^{N-1}}{(N-1)!}\tfrac{\partial^{N-1}K_{rp}}{\partial y^{N-1}}|_{(x,c)},\ r,p=1,2,...,s. \end{aligned} \tag{7}$$

Substituting the equation (7) in to (6), we get the following system:

$$u_r^N(x)=f_r(x)+\lambda\sum_{p=1}^{s}\sum_{l=1}^{N}\lambda_l^{rp}(x)\int_\Gamma \mu_l(y)u_p^N(y)dy,\ r=1,2,...,s, \tag{8}$$

where

$$\begin{aligned} \lambda_l^{rp}(x) &= \frac{1}{(l-1)!}\frac{\partial^{l-1}K_{rp}}{\partial y^{l-1}}|_{(x,c)},\ r,p=1,2,...,s, \\ \mu_l(y) &= (y-c)^{l-1},\ l=1,2,...,N. \end{aligned}$$

Let

$$\beta_l^{(p)}=\int_\Gamma \mu_l(y)u_p^N(y)dy,\ l=1,2,...,N,\ p=1,2,...,s,$$

which are unknown constants depending on $u_p^N(y)$. So equation (8) can be rewritten as follows:

$$u_r^N(x)=f_r(x)+\lambda\sum_{p=1}^{s}\sum_{l=1}^{N}\beta_l^{(p)}\lambda_l^{rp}(x),\ r=1,2,...,s. \tag{9}$$

By multiplying (9) with $\mu_j(x)$, $j=1,2,...,N$, and integrating, we obtain

$$\begin{gathered} \textstyle\int_\Gamma \mu_j(x)u_r(x)dx=\int_\Gamma \mu_j(x)f_r(x)dx+\lambda\sum\limits_{p=1}^{s}\sum\limits_{l=1}^{N}\beta_l^{(p)}\int_\Gamma \mu_j(x)\lambda_l^{rp}(x)dx. \\ r=1,2,...,s,\ j=1,2,...,N. \end{gathered} \tag{10}$$

Let

$$F^{(r)}=[F_1^{(r)},F_2^{(r)},...,F_s^{(r)}]^T,\ r=1,2,...,N, \tag{11}$$

with

$$F_j^{(r)}=\textstyle\int_\Gamma \mu_j(x)f_r(x)dx,\ r=1,2,...,s,\ j=1,2,...,N, \tag{12}$$

and

$$\beta^{(r)} = [\beta_1^{(r)},\beta_2^{(r)},...,\beta_N^{(r)}]^T,\ r=1,2,...,s.$$

We shall rewrite equation (10) in the matrix form

$$\beta^{(r)}=F^{(r)}+A_1^{(r)}\beta^{(1)}+A_2^{(r)}\beta^{(2)}+...+A_s^{(r)}\beta^{(s)},\ r=1,2,...,s, \tag{13}$$

where

$$A_p^{(r)} = [\lambda a_{ij}]_{1\le i,j\le N},\ a_{ij}=\int_\Gamma \mu_i(x)\lambda_j^{rp}(x)dx.$$

Or more generally one can write the system (13) as

$$A\beta=F, \tag{14}$$

with

$$A=\begin{bmatrix} I-A_1^{(1)} & A_2^{(1)} & ... & A_s^{(1)} \\ A_1^{(2)} & I-A_2^{(2)} & ... & A_s^{(2)} \\ \vdots & \vdots & \ddots & \vdots \\ A_1^{(s)} & A_2^{(s)} & \cdots & I-A_s^{(s)} \end{bmatrix},$$

$$\beta=\begin{bmatrix} \beta^{(1)} \\ \beta^{(2)} \\ \vdots \\ \beta^{(s)} \end{bmatrix}, \qquad F=\begin{bmatrix} F^{(1)} \\ F^{(2)} \\ \vdots \\ F^{(s)} \end{bmatrix},$$

where A and F, are block known matrices of $Ns \times Ns$ and $Ns \times 1$ dimensions respectively, I is $N \times N$ identity matrix and β is unknown $Ns \times 1$ block matrix to be determined. The algebraic linear system (14) can be solved exactly or numerically. Substituting the solution of system (14) in the equation (9), the approximate solution of the system of integral equations (1) is obtained as follows:

$$\mathcal{U}^N(x)=\mathcal{F}(x)+\lambda\mathcal{A}(x)\beta, \tag{15}$$

with

$$\mathcal{F}(x)=\begin{bmatrix} f_1(x) \\ f_2(x) \\ \vdots \\ f_s(x) \end{bmatrix}, \qquad \mathcal{A}(x)=\begin{bmatrix} \mathcal{A}_1^{(1)}(x) & \mathcal{A}_2^{(1)}(x) & ... & \mathcal{A}_s^{(1)}(x) \\ \mathcal{A}_1^{(2)}(x) & \mathcal{A}_2^{(2)}(x) & ... & \mathcal{A}_s^{(2)}(x) \\ \vdots & \vdots & \ddots & \vdots \\ \mathcal{A}_1^{(s)}(x) & \mathcal{A}_2^{(s)}(x) & \cdots & \mathcal{A}_s^{(s)}(x) \end{bmatrix},$$

where

$$\mathcal{A}_p^{(r)}(x)=[\Lambda_1(x),\Lambda_2(x),...,\Lambda_N(x)], \qquad \Lambda_i(x)=\lambda_i^{rp}(x),\ i=1,2,...,N,\ r,p=1,2,...,s.$$

3. Convergence analysis

In this section, the convergence analysis of the new method is investigated. From Section 2 and Taylor's theorem, we have

$$K_{ij}(x,y)=\sum_{l=1}^{N}\mu_l(y)\lambda_l^{ij}(x)+R_{ij}^N(x,y),\ i,j=1,2,...,s, \tag{16}$$

with

$$R_{ij}^N(x,y)=\frac{(y-c)^N}{(N)!}\frac{\partial^N K_{ij}}{\partial y^N}|_{(x,\xi_{ij}^N)},$$

where ξ_{ij}^N is between y and c. In addition,

$$\lim_{N\to\infty}R_{ij}^N(x,y)=0,\ \forall y\in(c-\delta_{ij},c+\delta_{ij}), \tag{17}$$

where δ_{ij} is convergence radius of Taylor-series of K_{ij} to variable y about $y=c$. The following proposition is explicit result of the Taylor-series expansion and it is easy to prove.

Proposition 3.1. *Let* $\max\limits_{x\in[0,1]}|\frac{\partial^N K_{ij}}{\partial y^N}|_{(x,\xi_{ij}^N)}|=M$, *then*

$$\min_{c\in[0,1]}\max_{(x,y)\in\Omega}|R_{ij}^N(x,y)|=\frac{M}{2^N N!},$$

where $\Omega=[0,1]\times[0,1]$.

Now let $\mathcal{X}$ be a Banach space with norm $\| \mathcal{U} \|_2 = \sup_{x\in\Gamma} \sqrt{\sum_{i=1}^{s} | u_i(x) |^2}$. Similar to (2), equation (8) can be rewritten as the following operator form:

$$(I - \lambda \mathbb{K}^N)\mathcal{U}^N = \mathcal{F}, \tag{18}$$

where $\mathbb{K}^N$ be the approximate integral operator associated with matrix kernel

$$\mathcal{K}^N(x,y) = [K_{ij}^N(x,y)]_{1\le i,j\le s},\ K_{ij}^N(x,y) = \sum_{l=1}^{N} \lambda_l^{ij}(x)\mu_l(y),\ i,j = 1,2,...,s.$$

We denote the error expression by

$$\mathcal{E}(x) = \mathcal{U}^N(x) - \mathcal{U}(x), \tag{19}$$

where $\mathcal{U}^N(x)$ and $\mathcal{U}(x)$ are approximate and exact solution of the system of integral equations, respectively. To show the convergence of the proposed method, we first prove the following lemma.

Let $Y = \{\mathcal{K} : \mathbb{R}^2 \to \mathbb{R}^{s\times s} | \mathcal{K} \in C(\Gamma\times\Gamma)\}$, we define the following norm on Y which is used in convergence analysis of new method

$$||\mathcal{K}||_F = \max_{(x,y)\in\Gamma\times\Gamma} \sqrt{\sum_{i=1}^{s}\sum_{j=1}^{s} |K_{ij}(x,y)|^2}.$$

Lemma 3.1. *Let $\mathbb{K} : \mathcal{X} \to \mathcal{X}$ be the bounded operator (2), and $\mathbb{K}^N : \mathcal{X} \to \mathcal{X}$ is the sequence of integral operator (18), then*

$$|||\mathbb{K} - \mathbb{K}^N||| \to 0 \ as\ N \to \infty, \tag{20}$$

where

$$|||\mathbb{K} - \mathbb{K}^N||| := \max_{\|\mathcal{U}\|_2=1} \| (\mathbb{K} - \mathbb{K}^N)\mathcal{U} \|_2 .$$

Proof. By using the properties of the integral norm and consistency of two norms $||.||_F$ and $||.||_2$, we have

$$\begin{aligned} |||\mathbb{K} - \mathbb{K}^N||| &= \max_{x\in\Gamma, \|\mathcal{U}\|_2=1} \| \int_\Gamma (\mathcal{K}(x,y) - \mathcal{K}^N(x,y))\mathcal{U}(y)dy \|_2 \\ &\le \max_{x\in\Gamma} \int_\Gamma \| (\mathcal{K}(x,y) - \mathcal{K}^N(x,y)) \|_F \, dy. \end{aligned}$$

From (16), (17), we have

$$\| (\mathcal{K}(x,y) - \mathcal{K}^N(x,y)) \|_F = \| R^N(x,y) \|_F \to 0 \ as\ N \to \infty,$$

where $R^N(x,y) = [R_{ij}(x,y)]_{1\le i,j\le s}$, which completes the proof. □

We note that the proposed method is included methods which takes a sequence of approximation problems that, their solutions converges to the solution of exact problem (perturbation theory). The following theorem describes a general convergence criterion for this class of method [4].

Theorem 3.1. *Let V and W be normed spaces with at least one of them being complete. Assume $L \in \mathcal{L}(V,W)$ has a bounded inverse $L^{-1} : W \to V$. Assume $M \in \mathcal{L}(V,W)$ satisfies*

$$|||M - L||| < \frac{1}{|||L^{-1}|||}. \tag{21}$$

Then $M : V \to W$ is a bijection, $M^{-1} \in \mathcal{L}(W,V)$ and

$$|||M^{-1}||| \le \frac{|||L^{-1}|||}{1 - |||L^{-1}|||.|||L - M|||}. \tag{22}$$

Moreover

$$|||L^{-1} - M^{-1}||| \leq \frac{|||L^{-1}|||^2.|||L - M|||}{1 - |||L^{-1}|||.|||L - M|||}. \tag{23}$$

For the solutions of the equations $Lv_1 = w$ and $Mv_2 = w$, we have the estimate

$$\| v_1 - v_2 \| \leq |||M^{-1}|||. \| (L - M)v_1 \| . \tag{24}$$

For the convergence property of the proposed method, we apply the above result to obtain the following main theorem. □

Theorem 3.2. *Assume that $\mathbb{K} : \mathcal{X} \to \mathcal{X}$ be the bounded operator (2) and right hand-side $\mathcal{F}$ be a bounded vector function with respect to $||.||_2$. If $(I - \lambda\mathbb{K}) \in \mathcal{L}(\mathcal{X}, \mathcal{X})$ has a bounded inverse $(I - \lambda\mathbb{K})^{-1} : \mathcal{X} \to \mathcal{X}$, then $(I - \lambda\mathbb{K}^N) : \mathcal{X} \to \mathcal{X}$ is a bijection, $(I - \lambda\mathbb{K}^N)^{-1} \in \mathcal{L}(\mathcal{X}, \mathcal{X})$ and*

$$|||(I - \lambda\mathbb{K}^N)^{-1}||| \leq \frac{|||(I - \lambda\mathbb{K})^{-1}|||}{1- \mid \lambda \mid .|||(I - \lambda\mathbb{K})^{-1}|||.|||\mathbb{K} - \mathbb{K}^N|||}, \tag{25}$$

where $(I - \lambda\mathbb{K}^N)$ is the operator (18). Moreover for any sufficiently large N $(N > \mathcal{M})$ the solution of system (18) converges to the solution of system (2).

Proof. According to Lemma 3.2, we have

$$|\lambda|.|||\mathbb{K} - \mathbb{K}^N||| \leq \frac{1}{|||(I - \lambda\mathbb{K})^{-1}|||},$$

for some large N. By taking

$$L = I - \lambda\mathbb{K}, \ M = I - \lambda\mathbb{K}^N,$$

in Theorem 3.1, the proof is completed. □

4. Numerical experiments

In this section, some numerical examples are presented to illustrate the effectiveness of proposed method for solving system of Fredholm integral equations (1). We denote $e_i, \ i = 1, 2, \ldots, s$ the maximum error $e_i = \max_{0 \leq j \leq n} |u_i^N(t_j) - u_i(t_j)|$, on the points $t_j = j\Delta t, \ j = 0, \ldots, n$ with $\Delta t = 0.001$ and $E_i, \ i = 1, 2, \ldots, s$ the error function $E_i(x) = |u_i^N(x) - u_i(x)|, \ x \in \Gamma$, where $u_i^N(x)$ and $u_i(x)$ are the ith element of approximate and exact solutions of (1), respectively. Numerical results of error values are listed in Tables 1, 2 and 3. In these tables the notations c and N are the same as the one introduced in the Section 2 and $cond(A)$ is denoted to the condition number of matrix A in (14). As is illustrated in Table 3, the computed error values e_1 and e_2 for $c = 0.5$ are better than those that for $c = 0$. This is due to Proposition 1. Convergence history of the error functions $E_i(x), i = 1, \ldots, s$, for Examples 4.1, 4.2 and 4.3 are shown in Figures 1 and 2. These figures show the fast convergence rate of the new method for some small integer N. All examples presented in this section were computed in double precision with a MATLAB code. In addition, we used the *quad* and *diff* MATLAB functions to approximate the involved definite integrals and to compute the required derivatives, respectively. Also, the resulting block algebraic linear system of equations obtained by the new method were computed exactly. However, one can compute it numerically for large system.

Example 4.1. Consider the system of linear Fredholm integral equations

$$\begin{cases} u_1(x) &= f_1(x) - \int_0^1 e^{xy} u_1(y)dy - \int_0^1 cos(xy) u_2(y)dy, \\ u_2(x) &= f_2(x) - \int_0^1 e^{x^2 y} u_1(y)dy - \int_0^1 e^{xy} u_2(y)dy, \end{cases}$$

Table 1. Numerical results for Example 4.1 with $c = 0$.

N	e_1	e_2	$cond(A)$
3	$9.80891E-02$	$1.22840E-01$	3.84335
6	$3.01207E-04$	$6.54223E-04$	4.05413
9	$6.50708E-07$	$9.18434E-07$	4.14717
12	$5.46298E-10$	$5.36826E-10$	4.19857
15	$1.27454E-13$	$1.75859E-13$	4.23125

Table 2. Numerical results for Example 4.2 with $c = 0$.

N	e_1	e_2	$cond(A)$
3	$1.01818E-01$	$1.01818E-01$	3.74787
6	$5.22163E-04$	$5.22163E-04$	4.00881
9	$7.57749E-07$	$7.57799E-07$	4.14836
12	$5.02640E-10$	$5.52640E-10$	4.22836
15	$1.20017E-10$	$2.14122E-11$	4.28002

Table 3. Numerical results for Example 4.3 with $c = 0$ and $c = 0.5$.

	c=0		c=0.5	
N	e_1	e_2	e_1	e_2
3	$5.98494E-01$	$1.59502E-01$	$5.90905E-2$	$1.93954E-2$
6	$1.24752E-01$	$2.86340E-02$	$8.93282E-3$	$1.91431E-3$
9	$4.11875E-03$	$7.80963E-04$	$1.29482E-05$	$2.36340E-6$
12	$5.80786E-5$	$9.46699E-6$	$8.15169E-08$	$1.26105E-08$
15	$4.30345E-7$	$6.17140E-8$	$2.94214E-11$	$4.05631E-12$
18	$1.87822E-9$	$2.40950E-10$	$4.55191E-14$	$6.97774E-15$

where $f_1(x) = e^x + \frac{sin(x)}{x} + \frac{e^{x+1}-1}{x+1}$ and $f_2(x) = \frac{e^x-1}{x} + \frac{e^{x^2+1}-1}{x^2+1} + 1$ with exact solutions $(u_1(x), u_2(x)) = (e^x, 1)$.

Example 4.2. Consider the system of linear Fredholm integral equations

$$\begin{cases} u_1(x) &= f_1(x) - \int_0^1 e^{xy} u_1(y)dy - \int_0^1 e^{xy} u_2(y)dy, \\ u_2(x) &= f_2(x) - \int_0^1 e^{x^2y} u_1(y)dy - \int_0^1 e^{x^2y} u_2(y)dy, \end{cases}$$

where $f_1(x) = e^x + \frac{e^{x+1}-1}{x+1} + \frac{e^{x-1}-1}{x-1}$ and $f_1(x) = e^{-x} + \frac{e^{x^2+1}-1}{x^2+1} + \frac{e^{x^2-1}-1}{x^2-1}$ with exact solutions $(u_1(x), u_2(x)) = (e^x, e^{-x})$.

Example 4.3. Consider the system of linear Fredholm integral equations [16],

$$\begin{cases} u_1(x) &= f_1(x) - \int_0^1 e^{x-y} u_1(y)dy - \int_0^1 e^{(x+2)y} u_2(y)dy, \\ u_2(x) &= f_2(x) - \int_0^1 e^{xy} u_1(y)dy - \int_0^1 e^{x+y} u_2(y)dy, \end{cases}$$

where $f_1(x) = 2e^x + \frac{e^{x+1}-1}{x+1}$ and $f_2(x) = e^x + e^{-x} + \frac{e^{x+1}-1}{x+1}$ with exact solutions $(u_1(x), u_2(x)) = (e^x, e^{-x})$.

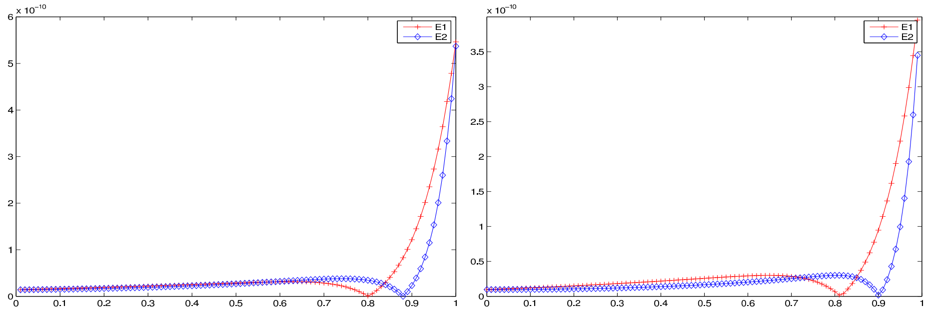

FIGURE 1. Error functions $E_i(x), i = 1, 2$, for Example 4.1 (left) and Example 4.2 (right) with $c = 0$ and $N = 12$.

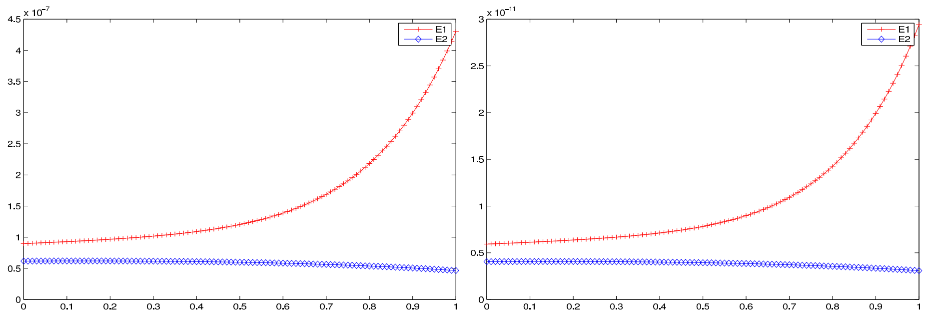

FIGURE 2. Error functions $E_i(x), i = 1, 2$, for Example 4.3 with $c = 0$(left), $c = 0.5$(right) and $N = 15$.

5. Conclusion

A new method for solving system of linear Fredholm integral equations of second kind was proposed. The method is based on the Taylor expansion of each kernel. This lead to a system of Fredholm equations having degenerate kernels that is reduced to a block linear algebraic system. By solving the block linear system, an approximate solution was obtained for the system of integral equations. Under certain conditions, it was shown that the approximate solution obtained by the proposed method converges to the exact solution. Numerical results verified the efficiency and robustness of the proposed method.

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Saeed Karimi is an assistant Professor of applied mathematics and academic member of Persian Gulf University, Bushehr, Iran. His research interests are mainly iterative methods and preconditioning for solving largew sparse linear system of equations, numerical methods for solving integral equations. He received his B.Sc. degree in applied mathematics in 1995 from Shiraz University, Shiraz, Iran and M.Sc. degree in applied mathematics in 1997 from Iran University of Science and Technology, Tehran, Iran. Also he received his Ph.D. degree in 2006, under supervision of Prof. Faezeh Toutounian at Ferdowsi University of Mashhad, Mashhad, Iran.

Meisam Jozi is a Ph.D. student in Department of Mathematics, Persian Gulf University, Bushehr, Iran. His research interests are iterative methods for solving large sparse linear system of equations and integral equations. He received his B.Sc. degree in applied mathematics in 2008 from Payame Noor University, Firoozabad, Iran and M. Sc. degree in applied mathematics in 2010 from Persian Gulf University, Bushehr, Iran.