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ON CLOSED MAPPINGS OF UNIFORM SPACES

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Abstract. In the paper the $u-$continuous, $u-$closed, z_u-closed, $u-$perfect mappings have been determined and their some properties have been established. The importance of these mappings classes is caused by that $u-$closed mappings are a subclass of the closed mappings class, and the closed mappings class is a subclass of the z_u-closed mappings.

Keywords: open (closed) sets, zero− (cozero)sets, uniformly continuous mapping, perfect mapping, bicompact.

AMS Subject Classification: 54C05, 54C10, 54C30, 54C35, 54C45, 54C50.

1. Introduction

Z. Frolik ([4]) introduced z-closed mappings, which are a natural generalization of the closed mappings ([3]).

Definition 1.1. ([4]). *A continuous mapping $f : X \to Y$ of a topological space X into a topological space Y is called $z-$closed, if the image $f(F)$ of any functionally closed ($\equiv$ zero-set) F in X is closed set in Y.*

Below the uniform analogues of closed and $z-$ closed mappings have been determined. Everywhere necessary information and denotations are taken from books [6] and [1],[2],[5].

Every uniform space be uX, where $\mathcal{U}$ be a uniformity in a uniform coverings terms, $f : uX \to vY$ be a mapping of uniform space uX into uniform space vY and if $f(F) = Y$, then the mapping f is surjective. We denote $C^*(uX)$ to be a ring of all bounded uniformly continuous functions on uX, $\mathfrak{Z}(uX) = \left\{f^{-1}(0) : f \in C^*(uX)\right\}$ be a set of all uniformly zero-sets ($\equiv$ uniformly closed sets ([2]), $\mathcal{L}(uX) = \left\{f^{-1}(\mathbb{R}\setminus\{0\}) : f \in C^*(uX)\right\}$ be a set of all uniformly cozero-sets ($\equiv$ uniformly open sets ([2])) of the uniform space uX.

Let $u_{\mathbb{R}}\mathbb{R}$ be a set of real numbers $\mathbb{R}$ with natural uniformity $\mathcal{U}_{\mathbb{R}}$, generated by the metrics $\rho(x, y) = |x - y|$ for any $x, y \in \mathbb{R}$, and $u_I I$ be a segment $I = [0, 1]$ with uniformity $\mathcal{U}_I$, induced by the uniformity $\mathcal{U}_{\mathbb{R}}$.

Definition 1.2. ([2]). *A mapping $f : uX \to vY$ is called $u-$continuous, if the inverse image $f^{-1}(F) \in \mathfrak{Z}(uX)(f^{-1}(U) \in \mathcal{L}(uX))$ for any $F \in \mathfrak{Z}(uY)(U \in \mathcal{L}(uY))$.*

Remark 1.1. *Every uniformly continuous mapping $f : uX \to vY$ is $u-$continuous. If $\mathcal{U}_f$ and $\mathcal{V}_f$ are fine uniformities of Tychonoff spaces X and Y respectively, then for mapping $f : u_f X \to v_f Y$ u_f- continuity is equivalent to the continuity of mapping $f : X \to Y$: There is $u-$continuous mapping $f : uX \to vY$, which is not uniformly continuous.*

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Theorem 1.1. ([2]). *Let* $g_1^{-1}(0) = F_1 \in \mathfrak{Z}(uX)$ *and* $g_2^{-1}(0) = F_2 \in \mathfrak{Z}(uX)$, *where* $g_1, g_2 \in C^*(uX)$ *and* $F_1 \cap F_2 = \emptyset$. *Then the function* $f : uX \to u_I I$, *determined as* $f(x) = |g_1(x)|/(|g_1(x)| + |g_2(x)|)$ *for any* $x \in X$, *is a* $u-$*function.*

Example 1.1. *Let* $X = [-1; 0) \cup (0; 1]$ *and a uniformity* $\mathcal{U}$ *on* X *is induced by the uniformity* $\mathcal{U}_{\mathbb{R}}$ *of* $\mathbb{R}$. *The sets* $[-1; 0)$ *and* $(0; 1]$ *are no uniformly separated, hence, there is no uniformly continuous function on the uniform space* uX, *which is separates these sets. Functions* $g_i : uX \to u_{\mathbb{R}}\mathbb{R}$, $i = 1, 2$, *determined as* $g_1(x) = \rho(x, [-1; 0))$ *and* $g_2(x) = \rho(x, (0; 1])$ *are uniformly continuous. Then the function* $f(x) = g_1(x)/(g_1(x) + g_2(x))$ *is an example of the* $u-$*continuous function, which is not uniformly continuous.*

2. Main results

Example 2.1. *Let* $\varepsilon > 0$ *and* $\mathbb{R}^+ = (0; +\infty)$. *A uniformity* $\mathcal{U}_{\mathbb{R}}$ *of real numbers* $\mathbb{R}$ *is generated by the basis* $\mathcal{B}$, *consisting of uniform coverings* $\alpha_\varepsilon = \{O_\varepsilon(x) : x \in \mathbb{R}\}$, *where* $O_\varepsilon(x) = (x - \varepsilon, x + \varepsilon)$ *is open interval with center at point* x *of length* 2ε. *Let* $\mathcal{P}(\mathbb{R})$ *be a set of all finite subsets of* $\mathbb{R}$ *and* $\mathbb{R}^+$. *For any* $\varepsilon \in \mathbb{R}^+$ *and any* $A \in \mathcal{P}(\mathbb{R})$ *suppose* $\alpha_{\varepsilon,A} = \{O_\varepsilon(x) \setminus A : x \in \mathbb{R} \setminus A\} \cup \{\{a\} : a \in A\}$. *A family* $\mathcal{B}' = \alpha_{\varepsilon,A} : \varepsilon \in \mathbb{R}^+,\ A \in \mathcal{P}(\mathbb{R})$ *is a basis of some uniformity* $\mathcal{U}'$ *on* $\mathbb{R}$, *more strong, than uniformity* $\mathcal{U}_{\mathbb{R}}$. *Really,* $\alpha_{\varepsilon_1,A_1} \wedge \alpha_{\varepsilon_2,A_2} \succ \alpha_{\varepsilon,A_1\cup A_2}$, *where* $\varepsilon = \min\{\varepsilon_1, \varepsilon_2\}$ *and the covering* $\alpha_{\delta,A}$ *is starry inscribed to the covering* $\alpha_{\varepsilon,A}$, *where* $\delta = \frac{\varepsilon}{3}$. *We note, that* $\mathcal{U}_{\mathbb{R}} \subset \mathcal{U}'$ *and* $\mathcal{U}'$ *generates discrete topology on the* $\mathbb{R}$.

Proposition 2.1. *A set of rational numbers* $\mathbb{Q}$ *is not uniformly zero-set in the uniform space* $u'\mathbb{R}$, *i.e.* $\mathbb{Q} \notin \mathfrak{Z}(u'\mathbb{R})$.

Proof. We suppose, that $\mathbb{Q} \in \mathfrak{Z}(u'\mathbb{R})$, i.e. there is such uniformly continuous function $f \in C^*(u'\mathbb{R})$, that $\mathbb{Q}=f^{-1}(0)$. Then for any $n \in \mathbb{N}$ there exist $\varepsilon_n > 0$ and $A_n \in \mathcal{P}(\mathbb{R})$ such that the family $f(\alpha_{\varepsilon_n,A_n})$ is inscribed to the covering $\alpha_{\frac{1}{n}}$, i.e. for any $y \in O_{\varepsilon_n}(x)$ the formula $|f(x) - f(y)| < \frac{1}{n}$ is provided for all $x \in \mathbb{R}$. Let $x \notin A = \bigcup_{n=1}^{\infty} A_n$, in force of everywhere density of $\mathbb{Q}$ in $\mathbb{R}$, there is such $y \in \mathbb{Q} \setminus A$, that $|x - y| < \varepsilon_n$, hence for all $x \in \mathbb{R}$ for all such $x \notin A$ and $|x - y| < \varepsilon_n$ we have $|f(x)| < \frac{1}{n}$ for any $n \in \mathbb{N}$, i.e. $f(x) = 0$ for any $x \in \mathbb{R} \setminus A$. Thus, $\mathbb{R} \setminus A = f^{-1}(0)$, i.e. $\mathbb{R}=\mathbb{Q} \cup A$ is contradiction, since $\mathbb{Q}$ and A are countable sets, and $\mathbb{R}$ is uncountable. The proposition is proved. □

We consider the function $h : u'\mathbb{R} \to u_{\mathbb{R}}\mathbb{R}$, determined as $g(x) = 0$, if $x \in \mathbb{Q}$ and $g(x) = 1$, if $x \in \mathbb{R} \setminus \mathbb{Q}$. Then h is continuous function, which is not $u'-$continuous function, since $g^{-1}(0) = = \mathbb{Q} \notin \mathfrak{Z}(u'\mathbb{R})$. By means of example 2.1, it is naturally to determine a special closed mappings of uniform spaces.

Definition 2.1. *A mapping* $f : uX \to vY$ *is called* $u-$*closed if it is* $u-$*continuous and for any closed set* F *in* X *the image* $f(F)$ *is closed in* Y.

Definition 2.2. *A mapping* $f : uX \to vY$ *is called* z_u-*closed, if it is* $u-$*continuous and for any uniformly closed set* $F \in \mathfrak{Z}(uX)$ *the image* $f(F)$ *is closed in* Y.

Obviously, every $u-$closed mapping is z_u-closed. It takes place the next simple

Proposition 2.2. *Every* $u-$*closed mapping* $f : uX \to vY$ *is* z_u-*closed.*

Theorem 2.1. *A mapping* $f : uX \to vY$ *is* z_u-*closed if and only if for every point* $y \in Y$ *and every uniformly cozero-set* $U \in \mathcal{L}(uX)$, *containing* $f^{-1}(y)$, *i.e.* $f^{-1}(y) \subset U$, *there is such open neighborhood* V *of point* $y \in Y$, *that* $f^{-1}(V) \subset U$.

Proof. Necessity. Let the mapping $f : uX \to vY$ be a z_u-closed and $y \in Y$ be an arbitrary point and uniformly cozero-set $U \in \mathcal{L}(uX)$, containing $f^{-1}(y)$, i.e. $f^{-1}(y) \subset U$. Then $X \backslash U \in \mathfrak{Z}(uX)$ is uniformly zero-set and $f(X \backslash U)$ is closed in Y. Set $V = Y \backslash f(X \backslash U)$ is open in Y and $y \in V$, i.e. V is open neighborhood of point y. The next calculations: $f^{-1}(V) = f^{-1}(Y \backslash f(X \backslash U)) = X \backslash f^{-1}(f(X \backslash U)) \subset X \backslash (X \backslash U) = U$ are provided, i.e.$f^{-1}(V) \subset U$.

Sufficiency. Conversely, let the condition of theorem is provided: $F \in \mathfrak{Z}(uX)$ be an arbitrary uniformly zero-set. The set $U = X \backslash F \in \mathcal{L}(uX)$ is uniformly cozero-set and for any $y \in Y \backslash f(F)$ we have $f^{-1}(y) \subset X \backslash f\left(f^{-1}(f(F))\right) \subset X \backslash F = U$. Then there is an open neighborhood V_y of point $y \in Y \backslash f(F)$ such, that $f^{-1}(V_y) \subset U$. Suppose $V = \cup\{V_y : y \in Y \backslash f(F)\}$. Then V is open in Y and $Y \backslash f(F) \subset V$ and $f^{-1}(V) \subset U$, i.e. $f^{-1}(V) \cap F = \emptyset$. Then $V \cap f(F) = \emptyset$, i.e. $V \subset Y \backslash f(F)$. Consequently, $f(F) = Y \backslash V$, i.e. the set $f(F)$ is closed. The theorem is proved completely. □

The next theorem demonstrates, when z_u-closeness of mappings implies $u-$closeness.

Theorem 2.2. *If a mapping $f : uX \to vY$ is z_u-closed and $f^{-1}(y)$ is Lindelöf for any point $y \in Y$, then the mapping f is $u-$closed.*

Proof. Let $y \in Y$ be an arbitrary point, $f^{-1}(y)$ be a Lindelőf and U be an arbitrary open set, containing $f^{-1}(y)$, i.e. $f^{-1}(y) \subset U$. Family $\mathcal{L}(uX)$ is a basis of topology of the uniform space uX ([2]), hence for any point $x \in f^{-1}(y) \subset U$ there exists such uniformly cozero-set $V_x \in \mathcal{L}(uX)$, which is the open neighborhood of x, then $x \in V_x \subset U$. Then the family $\left\{V_x : x \in f^{-1}(y)\right\}$ is open covering of Lindelöf space $f^{-1}(y)$. Let $\{V_{x_n} : n \in \mathbb{N}\}$ be a countable subcovering. Since $V_{x_n} \in \mathcal{L}(uX)$ for all $n \in \mathbb{N}$, then $\mathcal{V}' = \cup\{V_{x_u} : n \in N\}$ is uniformly cozero-set ([2]) and $f^{-1}(y) \subset U' \subset U$. By z_u-closeness of mapping $f : uX \to vY$, there is such open neighborhood V of point $y \in Y$, that $f^{-1}(V) \subset U' \subset U$. Then, on one of the closed mappings criterion ([3]), it follows, that the mapping $f : uX \to vY$ is $u-$closed. □

The theorem is proved.

Corollary 2.1. *Let $f : uX \to vY$ be a bicompact $u-$continuous mapping, i.e. $f^{-1}(y)$ is bicompact for any $y \in Y$. Then the next conditions are equivalent:*

(1) *$f : uX \to vY$ is z_u-closed.*
(2) *$f : uX \to vY$ is $u-$closed.*

Proof. (1$\Longrightarrow$2). It follows immediately from the Theorem 2.2.
(2$\Rightarrow$1) It follows from the Proposition 2.2.
Corollary 2.1. allows to define a special perfect mappings. □

Definition 2.3. *A mapping $f : uX \to vY$ is called $u-$perfect, if it is $u-$ closed and bicompact.*

Remark 2.1. *Obviously, every uniformly perfect mapping* ([1]) *$f : uX \to vY$ is $u-$perfect, and every $u-$perfect mapping $f : uX \to vY$ is perfect.*

Example 2.2. *Let X be a locally bicompact Tychonoff space and aX its one-point Alexandroff bicompactification. Let $\mathcal{U}_f$ be a fine uniformity on X, and $\mathcal{U}_a$ be a minimal precompact uniformity on X (see* [7], [6]*,Chapter II, Ex.10), then $\mathcal{U}_a \subset \mathcal{U}_f$ and $\mathcal{U}_a \neq \mathcal{U}_f$, as for the Samuel bicompactifications $(s_{u_f}X, s\mathcal{U}_f)$ and $(s_{u_a}X, s\mathcal{U}_a)$, we have $s_{u_f}X = \beta X$ is a Stone-Čech bicompactification and $s_{u_a}X = aX$ is the Alexandroff bicompactification. Obviously, $\beta X \neq aX$ (it is suppose that there is more than one uniformity on X). A identical mapping $1_x : \mathcal{U}_aX \to \mathcal{U}_fX$ is a topological homeomorphism, it is not $u-$continuous mapping.Thus, the class of perfect and closed mappings more wider than the class of $u-$perfect and $u-$closed mappings.*

The next properties of $u-$continuous mappings of the uniform spaces are take place.

Proposition 2.3. *A composition $g \circ f : uX \to wZ$ of $u-$continuous mappings $f : uX \to vY$ and $g : vY \to wZ$ is $u-$continuous mapping.*

Proof. Immediately follows from the definition of $u-$continuous mapping (Definition 1.2). □

Theorem 2.3. *If a composition $g \circ f : uX \to wZ$ of $u-$continuous mappings $f : uX \to vY$ and $g : vY \to wZ$ is z_u-closed mapping, then restriction $g\,|_{f(X)} : v'f(X) \to wZ$, where $\mathcal{V}' = \mathcal{V} \bigwedge f(X)$, is z_u-closed mapping.*

Proof. Let $N \in \mathfrak{Z}(v'f\,(X))$, i.e. N is a uniformly closed in $f(X)$. Then from the properties of the uniformly closed sets ([6]) it is follows there such $N' \in \mathfrak{Z}(vY)$ exists, that $N = N' \cap f(X)$. Then $f^{-1}(N') \in \mathfrak{Z}(uX)$ and $g \circ f : uX \to wZ$ is z_u-closed mapping by the condition of the theorem. We have $g|_{f(X)}(N) = g\big|_{f(X)}(N' \cap f\,(X)) = g\,(N' \cap f\,(X)) = (g \circ f)(f^{-1}\,(N'))$ and $g\,|_{f(X)})\,(N)$ is closed in Z. The theorem is proved. □

Corollary 2.2. *If a composition $g \circ f : uX \to wZ$ of $u-$continuous mappings $f : uX \to vY$ and $g : vY \to wZ$ is $u-$closed mapping, then restriction $g\,|_{f(X)} : v'f(X) \to wZ$, where $\mathcal{V}' = \mathcal{V} \wedge f(X)$, is $u-$closed mapping.*

Proof. Proof follows from the z_u- closeness of any $u-$ closed mapping (Proposition 2.4.). □

Proposition 2.4. *Let $f : uX \to vY$ be $u-$continuous mapping and $u'X'$ be a uniform subspace of uX. Then restriction $f|_{X'} : u'X' \to v'f(X')$, where $\mathcal{V}' = \mathcal{V} \wedge f\,(X')$, is $u-$continuous mapping too.*

Proof. Let F be a uniformly closed set in $f\,(X')$, i.e. $F \in \mathfrak{Z}(v'f\,(X'))$. Then there such function $f \in C^*\,(v'f\,(X'))$ exists, that $F = g^{-1}(0)$. By the Katetov Theorem ([7]) there such function $h \in C^*(vY)$ exists, that $h|_{f(X')} = g$. Then a function $h \circ f : uX \to u_{\mathbb{R}}\mathbb{R}$ is $u-$continuous and $(h \circ f)|_{X'} = g \circ f|_{X'}$. Hence we have $(g \circ f|_{X'})^{-1}\,(0) = \left(h \circ f\right)^{-1}\big|_{X'} = f^{-1}\left(h^{-1}\,(0)\right) \cap X' =$ $= f^{-1}\left(g^{-1}\,(0)\right) \in \mathfrak{Z}\,(u'X')$, where $f^{-1}(h^{-1}\,(0)) \in \mathfrak{Z}(uX)$ and $f^{-1}\left(g^{-1}\,(0)\right) \cap X' = f^{-1}(h^{-1}\,(0))$. The proposition is proved. □

Proposition 2.5. *Let $f : uX \to vY$ be z_u- closed mapping and $v'Y'$ be a uniform subspace of vY, where $\mathcal{V}' = \mathcal{V} \wedge Y'$ and $Y' \subset Y$. Then a mapping of restriction $f|_{f^{-1}(Y')} : u'f^{-1}(Y) \to v'Y'$, where $\mathcal{U}' = \mathcal{U} \wedge f^{-1}(Y')$, is z_u-closed mapping too.*

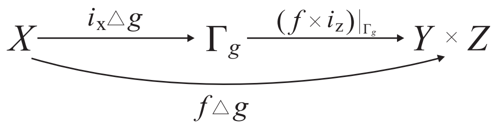
Proof. It follows from the equality $f|_{f^{-1}(Y')}\left(N \bigcap f^{-1}\,(Y')\right) = f(N) \bigcap Y'$ for any $N \in \mathfrak{Z}\,(X)$. Proposition is proved. □

Proposition 2.6. *Let $f : uX \to uY$ be $u-$closed mapping and $u'X'$ be closed uniform subspace of uX. Then a restriction $f|_{X'} : u'X' \to v'f(X')$, where $\mathcal{U}' = \mathcal{U} \wedge f(X)$, is a $u-$closed mapping too.*

Proof. It follows from the Proposition 2.13. and definition of $u-$closed mappings. □

Theorem 2.4. *Let $f : uX \to vY$ and $g : uX \to wZ$ be a surjective $u-$continuous mappings of the uniform spaces uX, vY, wZ and f is a $u-$closed mapping. Then diagonal product $f \triangle g : uX \to v \times wY \times Z$, where $\mathcal{V} \times \mathcal{W}$, is the product of the uniformities $\mathcal{V}$ and $\mathcal{W}$, is $u-$closed mapping.*

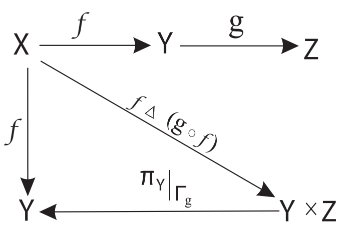
Proof. For a diagonal mapping $f \vartriangle g : uX \to v \times wY \times Z$, by the definition, we have $(f \vartriangle g)(x) = = (f(x), g(x))$. Let $i_X : uX \to uX$ and $i_Z : wZ \to wZ$ be identical uniform homeomorphisms. Suppose $f \times i_Z : u \times wX \times Z \to v \times wY \times Z$, $i_X \vartriangle g : uX \to u \times wX \times Z$, where $(f \times i_Z) : (x, z) = (f(x), z)$ and $(i_X \vartriangle g)(x) = (x, g(x))$ for any $x \in X$ and $z \in Z$. If $F \subset X$ and $M \subset Z$ are closed sets, then $f(F) \times M$ is closed subset of $Y \times Z$, hence, $(f \times i_Z)(F, M) = f(F) \times M$ and $f \times i_Z$ is $u-$closed mapping. The mapping $i_X \vartriangle g : X \to X \times Z$ is uniform homeomorphism of the space uX and $\Gamma_g = \{(x, g(x)) : x \in X\}$ is a graph of a mapping g, it is a closed subspace of $u \times wX \times Z$. The closeness of the graph Γ_g in $X \times Z$ follows from the uX and wZ are Hausdorf spaces. Then mapping $f \vartriangle g$ is a composition of the mappings $i_X \vartriangle g : uX \to u \times wX \times Z$ and $f \times i_Z|_{\Gamma_g} : v'\Gamma_g \to v \times wY \times Z$, where $\mathcal{V}' = \mathcal{V} \times \mathcal{W} \wedge \Gamma_g$ and mapping $(f \times i_Z)|_{\Gamma_g}$ is $u-$closed as a restriction of the closed mapping $f \times i_Z$ onto the closed subspace $\Gamma_g \subset X \times Z$, and $i_X \vartriangle g : uX \to v'\Gamma_g$ is a uniform homeomorphism. Thus,$f \vartriangle g = (f \times i_Z)|_{\Gamma_g} \circ (i_X \vartriangle g)$ is a $u-$closed mapping. We have a diagram. The theorem is proved. □



Theorem 2.5. *Let $f : uX \to vY$ and $g : vY \to wZ$ are a surjective $u-$continuous mappings of the uniform spaces uX, vY, wZ and a composition $g \circ f : uX \to wZ$ is $u-$closed mapping. Then the mapping $f : X \to vY$ is $u-$closed too.*

Proof. By the condition of theorem $g \circ f : uX \to wZ$ is $u-$closed and $f : uX \to vY$ is a $u-$continuous mapping, according to the Theorem 2.16., $f \vartriangle (g \circ f) : uX \to u \times wX \times Z$ is a $u-$closed mapping. By the surjectivity of mappings f and $g \circ f$, we have $(f \vartriangle (g \circ f))(x) = = (f(x), (g \circ f)(x))$ for any $x \in X$. Then $\{(f(x), (g \circ f)(x)) : x \in X\} = \{f(x), g(f(x)) : x \in X\} = = \{(y, g(y)) : y \in Y\} = \Gamma_g$.

Obviously, that $(f \vartriangle (g \circ f))(x) = \{f(x), g(f(x)) : x \in X\} = \{(y, g(y)) : y \in Y\} = \Gamma_g$. The graph Γ_g is closed subspace $Y \times Z$ and the mapping $\pi_Y|_{\Gamma_g} : v'\Gamma_g \to vY$, where $\pi_Y : v \times wY \times Z \to vY$ and $\mathcal{V}' = \mathcal{V} \times \mathcal{W} \wedge \Gamma_g$, is uniform homeomorphic mapping. Then $f = \pi_Y|_{\Gamma_g} \circ (f \vartriangle (g \circ f)) : uX \to vY$ is $u-$closed mapping as a composition of the uniform homeomorphism $\pi_Y|_{\Gamma_g} : v'\Gamma_g \to vY$ and $u-$closed mapping $f \vartriangle (g \circ f) : uX \to u \times wX \times Z$. The next diagram takes place. We note, that the closeness of graph Γ_g in $Y \times Z$ is essential, as soon as for any



closed $F \subset X$, $(f \vartriangle (g \circ f))(F) = F'$ is closed in Γ_g, hence it is closed in $Y \times Z$ and its image $\pi_Y|_{\Gamma_g}(F')$ is closed in Y. It means, that $f(F) = \pi_Y|_{\Gamma_g}(F')$ and $f(F)$ is closed in Y,i.e. f is $u-$closed. The theorem is proved. □

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