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ON THE RESTORATION PROBLEM WITH DEGENERATED DIFFUSION

MARAT I. TLEUBERGENOV[1], GULMIRA T. IBRAEVA[2]

Abstract. The sufficient conditions are obtained on the restoration problems' solvability in a class of the stochastic differential Ito systems of the first order (with random disturbances from a class of Wiener processes and the diffusion degenerated with regard to a part of variables) on the given properties of a movement, when a control is included into the coefficient of drift, by separation method. The aspect of driving parameters is defined, ensuring sufficient conditions of the given integral manifold's existence of constructed equations' set in nonlinear and linear cases.

Keywords: stochastic differential equations, integral manifold, restoration problem.

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1. Introduction

The fundamentals of the theory and the common methods of inverse problems' solving of differential systems are developed in [1-3, etc.] for determined systems the equations of which are ordinary differential equations (ODE). So, in Erugin's article [1] the set of ODE which have the given integral curve is constructed. This work, afterwards, has appeared establishing in formation and development of the theory of inverse problems of systems' dynamics, described by ODE. The statement, classification of differential systems' inverse problems of and their solving in a class of ODE are stated in works [2,3].

In works [4-6] inverse problems of dynamics are considered at the additional supposition about presence of random disturbances from class of Wiener processes and, in particular, by the quasi-inversion method it is solved: 1) the basic inverse problem of dynamics - construction of the set of stochastic differential Ito equations of second order, possessing the given integral manyfold; 2) the problem of restoration of movement's equations - construction of the set of driving parameters which are going into in the given system of the stochastic differential Ito equations of second order, by the given integral manyfold; and 3) the problem of equations' closure of movement - construction of the set of second order's closing stochastic differential Ito equations by the given system of equations and the given integral manyfold.

In particular, the problem of stochastic differential equations' construction of second order

$$\ddot{x} = f(x, \dot{x}, t) + D(x, \dot{x}, t)u + \sigma(x, \dot{x}, t)\dot{\xi} \tag{1}$$

by the given set

$$\Lambda(t): \qquad \lambda(x, \dot{x}, t) = 0, \text{ where } \lambda \in R^m, \ \lambda = \lambda(x, \dot{x}, t) \in C^{121}_{x\dot{x}t}, \tag{2}$$

[1]Institute of Mathematics, Almaty, Kazakhstan
e-mail: marat207@mail.ru
[2]Military Institute of Air Defence's Forces T. Begeldinov, Aktobe, Kazakhstan
e-mail: gulmira_ibraeva@mail.ru

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so that the set (2) would be an integral manifold of equations (1), is considered in [6]. The system (1) can be interpreted as system of $2n$ equations of first order with diffusion degenerated exactly on n variables.

In a general view the system of the differential equations with degenerated diffusion of an aspect

$$\begin{cases} \dot{y} = f_1(y,z,v,w,t), \quad y \in R^{l_1}, \quad z \in R^{l_2}, \quad v \in R^{p_1}, \quad w \in R^{p_2}, \\ \dot{z} = f_2(y,z,v,w,t) + \sigma_1(y,z,v,w,t)\dot{\xi}, \quad \xi \in R^r, \\ \dot{v} = f_3(y,z,v,w,t) + L_1(y,z,v,w,t)u_1, \quad u_1 \in R^{k_1}, \quad u_2 \in R^{k_2}, \\ \dot{w} = f_4(y,z,v,w,t) + L_2(y,z,v,w,t)u_2 + \sigma_2(y,z,v,w,t)\dot{\xi}, \end{cases} \quad (3)$$

by given integral manifold

$$\Lambda(t): \quad \lambda(y,z,v,w,t) = 0, \quad where \quad \lambda = \lambda(y,z,v,w,t) \in C^{12121}_{yzvwt}, \lambda \in R^m. \quad (4)$$

was constructed earlier by authors in [7] by quasi-inversion method.

Here C^{12121}_{yzvwt} means the set of functions $\gamma(y,z,v,w,t)$, which are continuously differentiable on y, v and on t and doubly continuously differentiable on z, w; $l_1 + l_2 + p_1 + p_2 = n$; u_1, u_2 are components of control vector-function u; L_1, L_2 are matrices of dimension accordingly $(p_1 \times k_1)$, $(p_2 \times k_2)$.

Though quasi-inversion method gives theoretically necessary and sufficient conditions of solvability, i.e. the maximum wide set of the equations possessing given integral manifold is defined, but necessary and sufficient conditions of solvability, received by a quasi-inversion method, are no constructive, also their application in practice calls certain difficulties. Therefore development and application of other methods of research, developed in a class of the ODE [3], and obtaining of more constructive conditions of solvability in a class of the stochastic differential equations is of interest.

In particular, in the given article the stochastic inverse problem of restoration is solved by a separation method.

2. Stochastic problem with control on a drift

Let us give the system of the stochastic differential Ito equations of first order (3). It is required to define the vector-functions $u_1(y,z,v,w,t) \in R^{k_1}$ and $u_2(y,z,v,w,t) \in R^{k_2}$, $k_1 + k_2 = r$ entering into drift coefficient, by given integral manifold (4).

It is supposed, that f_1, f_2, f_3, f_4, σ_1, σ_2 belong to a class of functions K, which are continuous on t and Lipschitzian on y, z, v and w in a neighborhood of the set $\Lambda(t)$

$$U_h(\Lambda) = \{q = (y^T, z^T, v^T, w^T)^T : \rho(q, \Lambda(t)) < h, \quad h > 0\}. \quad (5)$$

The posed problem is full enough examined in [2,3] in case of lack of random disturbances $(\sigma_1 \equiv \sigma_2 \equiv 0)$.

Let us make the equation of disturbed movement for the solving of posed problem in view of build-up of a set of equations (3) by given integral manifold (4) by Ito rule of stochastic differentiation of complicated function [8, p.204].

$$\dot{\lambda} = \frac{\partial \lambda}{\partial t} + \frac{\partial \lambda}{\partial y} f_1 + \frac{\partial \lambda}{\partial z} f_2 + \frac{\partial \lambda}{\partial v} f_3 + \frac{\partial \lambda}{\partial w} f_4 + \frac{\partial \lambda}{\partial v} L_1 u_1 + \frac{\partial \lambda}{\partial w} L_2 u_2 + S_1 + S_2 + \frac{\partial \lambda}{\partial z}\sigma_1 \dot{\xi} + \frac{\partial \lambda}{\partial w}\sigma_2 \dot{\xi}, \quad (6)$$

where $S_1 = \frac{1}{2}\left[\frac{\partial^2 \lambda}{\partial z \partial z}\sigma_1 \sigma_1^T\right]$, $S_2 = \frac{1}{2}\left[\frac{\partial^2 \lambda}{\partial w \partial w} : \sigma_2 \sigma_2^T\right]$, and under $\left[\frac{\partial^2 \lambda}{\partial z \partial z} : D\right]$, following [8], it is understood the vector, the elements of which are the traces of matrices' products of corresponding elements' second derivatives $\lambda_\mu(y,z,v,w,t)$, $\mu = \overline{1,m}$ of vector $\lambda(y,z,v,w,t)$ on components

z on a matrix D:

$$\left[\frac{\partial^2\lambda}{\partial z\partial z}:D\right]=\left[\begin{array}{c} tr\left(\dfrac{\partial^2\lambda_1}{\partial z\partial z}D\right)\\ \vdots \\ tr\left(\dfrac{\partial^2\lambda_m}{\partial z\partial z}D\right)\end{array}\right].$$

And also it is introduced N.P. Erugin's type [1] an m- measured vector-function A and an $(m\times r)$-matrix B, possessing property $A(0;y,z,v,w,t)\equiv 0, B(0;y,z,v,w,t)\equiv 0$, also an equality

$$\dot{\lambda}=A(\lambda;y,z,v,w,t)+B(\lambda;y,z,v,w,t)\dot{\xi} \tag{7}$$

takes place.

Comparing the equations (6) and (7), we come to relations

$$\begin{cases} \dfrac{\partial\lambda}{\partial t}+\dfrac{\partial\lambda}{\partial y}f_1+\dfrac{\partial\lambda}{\partial z}f_2+\dfrac{\partial\lambda}{\partial v}f_3+\dfrac{\partial\lambda}{\partial w}f_4+\dfrac{\partial\lambda}{\partial v}L_1u_1+\dfrac{\partial\lambda}{\partial w}L_2u_2+S_1+S_2=A,\\ \dfrac{\partial\lambda}{\partial z}\sigma_1+\dfrac{\partial\lambda}{\partial w}\sigma_2=B, \end{cases}$$

which we will rewrite as follows

$$\begin{cases} \dfrac{\partial\lambda}{\partial v}L_1u_1+Du_2=A-\left(\dfrac{\partial\lambda}{\partial t}+\dfrac{\partial\lambda}{\partial y}f_1+\dfrac{\partial\lambda}{\partial z}f_2+\dfrac{\partial\lambda}{\partial v}f_3+\dfrac{\partial\lambda}{\partial w}f_4+S_1+S_2\right),\\ \dfrac{\partial\lambda}{\partial z}\sigma_1+\dfrac{\partial\lambda}{\partial w}\sigma_2=B, \end{cases} \tag{8}$$

where through D it is meant an $(m\times k_2)$ matrix $D=\dfrac{\partial\lambda}{\partial w}L_2$.

From the given relations it is necessary to discover u_1, u_2, σ_1, σ_2. For this purpose we use a separation method [3, p.21] of required system. Following a separation method, beforehand matrices D, $\dfrac{\partial\lambda}{\partial w}$, σ_2 and a vector-function u_2 we will present in an aspect:

$$D=(D',D''),\quad \frac{\partial\lambda}{\partial w}=(G',G''),\quad \sigma_2=\begin{pmatrix}\sigma_2'\\ \sigma_2''\end{pmatrix},\quad u_2=\begin{pmatrix}u_2'\\ u_2''\end{pmatrix},$$

where D' is a square matrix of dimensionality $(m\times m)$, D'' - $(m\times(k_2-m))$ matrix, G' is a square matrix of dimensionality $(m\times m)$, G'' - $(m\times(p_2-m))$ matrix, σ_2' - $(m\times r)$ matrix, σ_2'' - $((p_2-m)\times r)$ matrix, u_2' - m-vector, u_2'' - (k_2-m)-vector.

Then the system (24) can be noted in an aspect:

$$\begin{cases} \dfrac{\partial\lambda}{\partial v}L_1u_1+D'u_2'+D''u_2''=N,\\ \dfrac{\partial\lambda}{\partial z}\sigma_1+G'\sigma_2'+G''\sigma_2''=B, \end{cases} \tag{9}$$

where $N=A-\left(\dfrac{\partial\lambda}{\partial t}+\dfrac{\partial\lambda}{\partial y}f_1+\dfrac{\partial\lambda}{\partial z}f_2+\dfrac{\partial\lambda}{\partial v}f_3+\dfrac{\partial\lambda}{\partial w}f_4+S_1+S_2\right)$.

Let us suppose, that $det\ D'\neq 0$ and $det\ G'\neq 0$, then a solution of system (9) is possible to present in an aspect

$$u_2'=(D')^{-1}\big(N-\frac{\partial\lambda}{\partial v}L_1u_1-D''u_2''\big), \tag{10}$$

$$\sigma_2'=(G')^{-1}\big(B-\frac{\partial\lambda}{\partial z}\sigma_1-G''\sigma_2''\big). \tag{11}$$

Hence, the theorem is valid.

Theorem 2.1. *The set (4) is integral manifold of system of differential equations (3) if the following conditions are satisfied:*
1) square submatrices D', G' of matrices D, G are non-degenerate det $D' \neq 0$, det $G' \neq 0$;
2) under arbitrarily given u_1, $u_2'' \in K$ the first m coordinates u_2' of vector u_2 look like (10);
3) under arbitrarily given σ_1, $\sigma_2'' \in K$ a submatrix σ_2' of matrix σ_2 looks like (11).

3. The linear case of a stochastic problem with control on a drift

It is required on given linear on a drift stochastic differential Ito equation of the first order

$$\begin{cases} \dot{y} = D_1(t)y + D_2(t)z + D_3(t)v + D_4(t)w + d(t), \\ \dot{z} = C_1(t)y + C_2(t)z + C_3(t)v + C_4(t)w + c(t) + \sigma_1(t)\dot{\xi}, \\ \dot{v} = F_1(t)y + F_2(t)z + F_3(t)v + F_4(t)w + F_5(t)u_1 + f(t), \\ \dot{w} = G_1(t)y + G_2(t)z + G_3(t)v + G_4(t)w + G_5(t)u_2 + g(t) + \sigma_2(t)\dot{\xi} \end{cases} \tag{12}$$

to define control's vector-functions

$$u_1 = u(y, z, v, w, t) \quad \text{and} \quad u_2 = u(y, z, v, w, t) \in R^r$$

on the given linear integral manifold

$$\Lambda(t) : \lambda \equiv H_1(t)y + H_2(t)z + H_3(t)v + H_4(t)w + h(t) = 0. \tag{13}$$

The equation of a disturbed motion (6) in a considered problem looks like

$$\dot{\lambda} = E_1(t)y + E_2(t)z + E_3(t)v + E_4(t)w + E_5(t) + H_3(t)F_5(t)u_1 + H_4(t)G_5(t)u_2 + \\ + H_2(t)\sigma_1\dot{\xi} + H_4(t)\sigma_2\dot{\xi}, \tag{14}$$

where

$$E_1(t) = H_1(t)D_1(t) + H_2(t)C_1(t) + H_3(t)F_1(t) + H_4(t)G_1(t) + \dot{H}_1(t),$$

$$E_2(t) = H_1(t)D_2(t) + H_2(t)C_2(t) + H_3(t)F_2(t) + H_4(t)G_2(t) + \dot{H}_2(t),$$

$$E_3(t) = H_1(t)D_3(t) + H_2(t)C_3(t) + H_3(t)F_3(t) + H_4(t)G_4(t) + \dot{H}_3(t),$$

$$E_4(t) = H_1(t)D_4(t) + H_2(t)C_4(t) + H_3(t)F_4(t) + H_4(t)G_4(t) + \dot{H}_4(t),$$

$$E_5(t) = H_1(t)d(t) + H_2(t)c(t) + H_3(t)f(t) + H_4(t)g(t) + \dot{h}(t).$$

And, on the other hand, by means of any Erugin vector-function $A = A_1(t)\lambda$ and a matrix-function B_1 with the property $B_1(0, y, z, v, w, t) \equiv 0$ we have

$$\dot{\lambda} = A_1(t)\lambda + B_1(\lambda, y, z, v, w, t)\dot{\xi}. \tag{15}$$

The next equalities

$$\begin{cases} H_3(t)F_5(t)u_1 + H_4(t)G_5(t)u_2 = (A_1H_1 - E_1)\,y + (A_1H_2 - E_2)\,z + \\ \quad + (A_1H_3 - E_3)\,v + (A_1H_4 - E_4)\,w + (A_1h(t) - E_5)\,, \\ H_2\sigma_1 + H_4\sigma_2 = B_1 \end{cases} \tag{16}$$

follow from relations (14) and (15).

We will apply the separation method [3, p.21] for solving posed problem. Beforehand we will introduce the labels $M(t) = H_4G_5$, $\widetilde{N(t)} = (A_1H_1 - E_1)\,y + (A_1H_2 - E_2)\,z + (A_1H_3 - E_3)\,v + (A_1H_4 - E_4)\,w + (A_1h(t) - E_5)$ and, further, we will present the system (16) in an aspect

$$\begin{cases} M'u_2' = \widetilde{N} - H_3(t)F_5(t)u_1 - M''u_2'', \\ H_4'\sigma_2' = B_1 - H_2\sigma_1 - H_4''\sigma_2''. \end{cases} \tag{17}$$

where the matrices M, H_4, σ_2 and the vector-function $u_2(t)$ are divided into corresponding submatrices and corresponding vectors:

$$M = (M', M''), \quad H_4 = (H_4', H_4''), \quad \sigma_2 = (\sigma_2', \sigma_2''), \quad u_2 = \begin{pmatrix} u_2' \\ u_2'' \end{pmatrix},$$

where M' is a matrix has the dimensionality $(m \times m)$, $M''-(m \times (k_2 - m))$, $H_4'-(m \times m)$, $H_4''-(m \times (p_2 - m))$; $\sigma_2'-(m \times r)$, $\sigma_2''-$ $((p_2 - m) \times r)$; u_2' is an m-vector function, u_2'' is an $(r_2 - m)$-vector-function.

Let us suppose, that $det\ M' \neq 0$ and $det\ H_4' \neq 0$, then the relations

$$u_2' = \left(M'\right)^{-1}\left(\widetilde{N(t)} - H_3(t)F_5(t)u_1 - M''u_2''\right), \tag{18}$$

$$\sigma_2' = \left(H_4'\right)^{-1}\left(B_1 - H_2\sigma_1 - H_4''\sigma_2''\right), \tag{19}$$

follow from (17).

Hence, the following statement takes place:

Theorem 3.1. *The linear set (13) is integral manifold of differential equations' system linear on a drift (12) if the following conditions are satisfied:*
1) square submatrices M' and H_4 of the rectangular matrices $M = H_4G_5$ and H_4 possess property $detM' \neq 0$, $detH_4 \neq 0$;
2) at arbitrarily given u_1, $u_2' \in K$ the first m coordinates u_2' of the vector u_2 look like (18);
3) at arbitrarily given σ_1, $\sigma_2'' \in K$ the submatrix σ_2' of matrix σ_2 looks like (19).

4. Scalar case of the restoration problem

Let the system from four scalar stochastic differential Ito equations of first order is given

$$\begin{cases} \dot{x}_1 = g_1(x_1, x_2, x_3, x_4, t), \quad x = (x_1, x_2, x_3, x_4)^T \in R^4, \\ \dot{x}_2 = g_2(x_1, x_2, x_3, x_4, t) + \gamma_1(x_1, x_2, x_3, x_4, t)\dot{\zeta}, \quad \zeta inR^1, \\ \dot{x}_3 = g_3(x_1, x_2, x_3, x_4, t) + v_1(x_1, x_2, x_3, x_4, t)u_1, \quad u_1 \in R^1, \quad u_2 \in R^1, \\ \dot{x}_4 = g_4(x_1, x_2, x_3, x_4, t) + v_2(x_1, x_2, x_3, x_4, t)u_2 + \gamma_2(x_1, x_2, x_3, x_4, t)\dot{\zeta}. \end{cases} \tag{20}$$

It is required to define the scalar functions $u_1(x_1, x_2, x_3, x_4, t)$ and $u_2(x_1, x_2, x_3, x_4, t)$, entering into a drift coefficient, by given integral manifold

$$H(t): \quad \eta(x_1, x_2, x_3, x_4, t) = 0, \quad where \quad \eta = \eta(x_1, x_2, x_3, x_4, t) \in C^{1\ 2\ 1\ 2\ 1}_{x_1x_2x_3x_4\,t}, \eta \in R^1. \tag{21}$$

In other words, on given g_1, g_2, g_3, g_4, γ_1, γ_2 and η we will define control parameters u_1 and u_2 so, that the set (21) would be an integral manifold of the equation (20).

The disturbed movement's equation in a considered problem owing to a rule of Ito stochastic differentiation looks like

$$\dot{\eta} = \frac{\partial \eta}{\partial t} + \frac{\partial \eta}{\partial x_1}f_1 + \frac{\partial \eta}{\partial x_2}g_2 + \frac{\partial \eta}{\partial x_3}g_3 + \frac{\partial \eta}{\partial x_4}f_4 + \frac{\partial \eta}{\partial x_3}v_1u_1 +$$
$$+\frac{\partial \eta}{\partial x_4}v_2u_2 + \frac{1}{2}\frac{\partial^2 \eta}{\partial x_2^2}\gamma_1^2 + \frac{1}{2}\frac{\partial^2 \eta}{\partial x_4^2}\gamma_2^2 + \frac{\partial \eta}{\partial x_2}\gamma_1\dot{\zeta} + \frac{\partial \eta}{\partial x_4}\gamma_2\dot{\zeta}. \tag{22}$$

And also it is introduced N.P.Erugin type's [1] scalar functions a and b, possessing property $a(0; x_1, x_2, x_3, x_4, t) \equiv b(0; x_1, x_2, x_3, x_4, t) \equiv 0$, also an equality

$$\dot{\eta} = a(\eta; x_1, x_2, x_3, x_4, t) + b(\eta; x_1, x_2, x_3, x_4, t)\dot{\zeta} \tag{23}$$

takes place.

Comparing the equations (22) and (23), we come to relations

$$\begin{cases} \dfrac{\partial\eta}{\partial t}+\dfrac{\partial\eta}{\partial x_1}g_1+\dfrac{\partial\eta}{\partial x_2}g_2+\dfrac{\partial\eta}{\partial x_3}g_3+\dfrac{\partial\eta}{\partial x_4}g_4+\dfrac{\partial\eta}{\partial x_3}v_1u_1+\dfrac{\partial\eta}{\partial x_4}v_2u_2+\dfrac{1}{2}\dfrac{\partial^2\eta}{\partial x_2^2}\gamma_1^2+\dfrac{1}{2}\dfrac{\partial^2\eta}{\partial x_4^2}\gamma_2^2=a, \\ \dfrac{\partial\eta}{\partial x_2}\gamma_1+\dfrac{\partial\eta}{\partial x_4}\gamma_2=b, \end{cases}$$

which we will rewrite as follows

$$\begin{cases} \dfrac{\partial\eta}{\partial x_3}v_1u_1+\dfrac{\partial\eta}{\partial x_4}v_2u_2=M, \\ \dfrac{\partial\eta}{\partial x_2}\gamma_1+\dfrac{\partial\eta}{\partial x_4}\gamma_2=b, \end{cases} \tag{24}$$

where $M=a-\left(\dfrac{\partial\eta}{\partial t}+\dfrac{\partial\eta}{\partial x_1}f_1+\dfrac{\partial\eta}{\partial x_2}g_2+\dfrac{\partial\eta}{\partial x_3}g_3+\dfrac{\partial\eta}{\partial x_4}g_4+S_1+S_2\right)$.

Let us suppose, that $\frac{\partial\eta}{\partial x_4}\neq 0$ and $v_2\neq 0$, then a solution of system (24) is possible to present in an aspect

$$u_2=(\frac{\partial\eta}{\partial x_4}v_2)^{-1}\big(M-\frac{\partial\eta}{\partial v}v_1u_1\big), \tag{25}$$

$$\gamma_2=(\frac{\partial\eta}{\partial x_4})^{-1}\big(b-\frac{\partial\eta}{\partial x_2}\gamma_1\big). \tag{26}$$

Hence, the theorem is valid.

Theorem 4.1. *The set (21) is integral manifold of system of differential equations (20) if the following conditions are satisfied: 1)* $\dfrac{\partial\eta}{\partial x_4}\neq 0$, $v_2\neq 0$*; 2) under arbitrarily given* $u_1\in K$ *a function* u_2 *looks like (25); 3) under arbitrarily given* $\gamma_1\in K$ *a function* γ_2 *looks like (26).*

5. Conclusion

Thus, in general non-linear, general linear and scalar non-linear statements of stochastic restoration's problems with degenerated concerning a part of variables diffusion are solved by a separation method.

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Marat I. Tleubergenov was born in Kazakhstan in 1952. He received high education in mathematics from the Russian University of People's Friendship, Moscow in 1973. He got the Ph.D. degree from the Russian University of People's Friendship in 1983, Doctor of Sciences degree from the Institute of Mathematics in Almaty (Kazakhstan) in 1999. He is a head of dynamical systems' laboratory of the Institute of Mathematics, Almaty. He is an author of over 170 scientific works. His current research interests are in the areas of stochastic differential systems' inverse problems and stochastic stability of program movement.

Gulmira T. Ibraeva was born in Kazakhstan in 1968. She received high education in mathematics from the Aktobe State University, Kazakhstan in 1990. She got the Ph.D. degree from the Institute of Mathematics in Almaty (Kazakhstan) in 2008. She is an assistant professor at the Military Institute of Air Defense's Forces named T. Begeldinov, Aktobe, Kazakhstan. She is an author of over 20 scientific works.