TWMS J. Pure Appl. Math. V.7, N.1, 2016, pp.20-27

NAVIGATION OF THE WHEELED TRANSPORT ROBOT UNDER MEASUREMENT NOISE

VLADIMIR LARIN[1]

Abstract. The problem of navigation of the simple wheeled transport robot is considered. The problem is solved without use of accelerometers and gyros, but only by using measurement of kinematic parameters of movement. It is supposed, that the sensor of the angle of turn of the steering wheel has a regular error which is necessary to compensate. Correction of navigating data is carried out by means of signals of GPS. It is supposed, that GPS receiver may be in any point on a line connecting the middle of axes of forward and back wheels. The stated approach is based on the condition that the wheeled robot is considered as system with nonholonomic constraints. Efficiency of such navigating system is shown on the example.

Keywords: nonholonomic system, navigating system, GPS.

AMS Subject Classification: 93A30.

1. Introduction

It is possible to ascertain an urgency of problems of navigation and guidance of wheeled transport systems (see, for example [10] where there are further references). For navigation of such objects various systems are used, including: rather cheap inertial navigating systems (INS) which are corrected by signals of GPS [9], systems of stereoscopic machine vision [10], systems using the information about road [4], etc. It is essential, that at the engineering of the system, which are using INS and GPS the considerable attention is given to questions of detection of failures in such systems [9] or if it is possible, to questions of neutralisation of those or other errors of data units.

In [6] the problem of navigation of the simple wheeled transport robot (TR) has been investigated. It is essential, that the navigating problem in [6] was solved without use of accelerometres and gyros, i.e. without use INS, but only by use of the kinematic parameters of motion of TR (angle of rotation of a steering wheel, etc.). Correction of the calculated kinematic parameters of a motion was carried out by using GPS signals.

Both these procedures are based on the assumption, that TR was considered as system with nonholonomic constraints. So, for example, in the first case, for evaluation of angular velocity of TR was used the fact, that angular velocity is proportional to a tangent of an angle of rotation of a steering wheel. In the problem of correction, this model of TR allowed to use the information about projections of velocity, gained from GPS receiver, for reception of an estimate of TR orientation.

Lower, as well as in [6], the simplest problem of navigation of wheeled TR is considered. However, unlike [6], it is supposed, that:

[1]Institute of Mechanics of National Academy of Science of Ukraine, Kiev, Ukraine
e-mail: vblarin@mail.ru

Manuscript received June 2015.

1. GPS receiver is installed in any point on a line pairing the middle of axes of forward and back wheels;

2. The data unit of an angle of rotation of a steering wheel has a systematic error which is necessary to compensate.

The algorithm of neutralisation of such error is similar to algorithms in [2, 3, 7].

Efficiency of functioning of such navigating system is shown on the example.

2. Equations of motion

Let's gain (in the kinematic approach) equation of motion of TR with one steering wheel. Let the object makes a plainly-parallel motion in Oxy plane. Its standing is characterised by piece AB (figure 1).

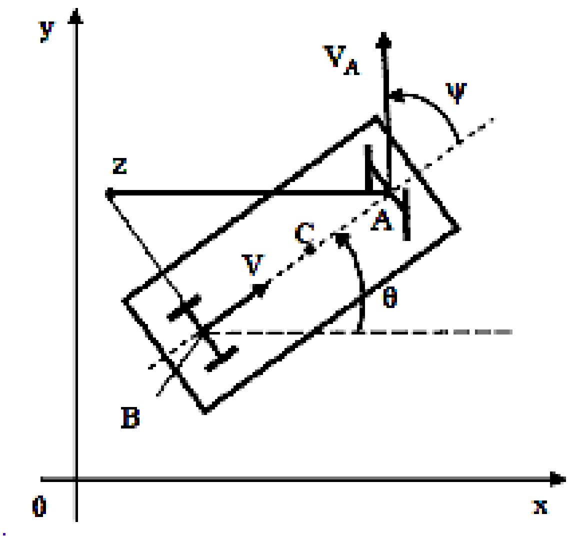

Figure 1. Kinematics' scheme of transport robot.

It is supposed, that velocity of point B is directed along piece AB, and velocity of point A makes angle ψ with a direction of piece AB (ψ – it is possible to interpret as angle of rotation of steering wheel). The standing of such system is spotted by co-ordinates (x, y) of point B and angle θ which forms piece AB and axis Ox. If to designate Z – the instantaneous centre of velocities of object, V_A, V – velocities of points A and B, $2L = |AB|$ ($|\cdot|$ – length of piece AB) it is possible to gain a following equation of motion of this object:

$$\dot{x} = V\cos\theta, \quad \dot{y} = V\sin\theta, \quad \dot{\theta} = (V/2L)\ tg\psi. \tag{1}$$

Thus, system (1) is describe, in the kinematic approach, a motion of the transport robot with one steering wheel (analogue of the equations [8, equations (11)]) when position of TR, are spotting by angle θ and co-ordinates of point B.

In more general case, if standing TR is spotted by corner θ and co-ordinates of point C (located on distance ℓ from the point B (figure 1)), analogues of the equations (1) look like:

$$\begin{bmatrix} V_x \\ V_y \end{bmatrix} = V \begin{bmatrix} -\frac{\ell}{2L} tg\psi & 1 \\ 1 & \frac{\ell}{2L} tg\psi \end{bmatrix} \begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix}, \quad \dot{\theta} = \left(\frac{V}{2L}\right) tg\psi. \tag{2}$$

In the equations (2) V_x, V_y – are velocities of point C.

It is obvious, that relations (2) at $\ell = 0$ is coincide with (1). We will note, that the first of relations (2) expressed the projections of velocity of point C through the projections of velocity of point B. However, it is possible to write down this relation and in the shape, allowing to express the projections of velocity of point B through the projections of velocity of point C:

$$V\begin{bmatrix} \sin\theta \\ \cos\theta \end{bmatrix} = A\begin{bmatrix} V_x \\ V_y \end{bmatrix}, \quad A = \frac{1}{a^2+1}\begin{bmatrix} -a & 1 \\ 1 & a \end{bmatrix}, \quad a = \frac{\ell}{2L}tg\psi. \tag{3}$$

In turn, the relation (3) allows to spot angle θ as a result of observation of projections V_x, V_y – velocities of point C in which GPS receiver is erected.

3. Navigation of TR

In [6], the possibility of creation of navigating complex of TR without use of accelerometers and gyros is shown. It is supposed, that figuring in (1), velocity V can be gained as a result of recording of angular velocity of back wheels of TR. As following from the last relation of (1), at known quantity of V, angular velocity $\dot{\theta}$ is spotted by quantity of angle of rotation of the steering wheel (quantity of ψ). In this situation, it is possible, by integration of (1), to find, as functions of time, co-ordinates of object (x, y) and object orientation (angle θ). Guessing, that recording of values of V, ψ occurs in the discrete moments of time, for reception the estimates of current values x, y, θ it is possible to use this or that approximation. So, let the measurement of parameters V and ψ occurs through equal intervals of time d, i.e. $t_{k+1} = t_k + d$, $k = 0, 1, 2, \ldots$, where t_k – is the moment of k- th measurement.

Let's designate u_k a vector of navigating parameters at the same time:

$$u_k = \begin{bmatrix} x_k & y_k & \theta_k \end{bmatrix}^T.$$

Hereinafter the upper index "T" means transposition.

The following approximation of non-linear system (1) which spots sequence of vectors u_k will be used further:

$$\begin{gathered} u_k = u_{k-1} + d(5w_k + 8w_{k-1} - w_{k-2})/12, \\ w_k = \begin{bmatrix} V_k\cos\bar{\theta}_k & V_k\sin\bar{\theta}_k & \frac{V_k}{2L}tg\psi_k \end{bmatrix}^T, \\ \bar{\theta}_k = \theta_{k-1} + \frac{dV_{k-1}tg\psi_{k-1}}{2L}. \end{gathered} \tag{4}$$

Let's note, that relations (4) can be viewed as analogue of the relation (1) [4]. Precision of such navigating systems (NS), in detail, are viewed in [6].

In case of system (2), the third component of vector w_k remains without changes, but the first two components of vector w_k ($w_k(1)$, $w_k(2)$) are spotted by relations which are following from (2):

$$\begin{bmatrix} w_k(1) \\ w_k(2) \end{bmatrix} = V\begin{bmatrix} -\frac{\ell}{2L}tg\psi_k & 1 \\ 1 & \frac{\ell}{2L}tg\psi_k \end{bmatrix}\begin{bmatrix} \sin\bar{\theta}_k \\ \cos\bar{\theta}_k \end{bmatrix}. \tag{5}$$

4. The filter equations

Let's designate $\delta\mu = \begin{bmatrix} \delta x & \delta y & \delta\theta & \delta\psi \end{bmatrix}^T$ a vector of errors of finding navigating parameters for the system which motion is described by the equations (1). Guessing, that velocity V is registered precisely, and $\delta\psi$ – the error of the data unit of the angle of rotation of the steering wheel is constant, it is possible to write down the equations of change of errors of the navigating system using as the initial information the kinematics parameters of system (1) (analogue of the equation (3.1) [7]):

$$\delta\dot{\mu} = F\delta\mu + \eta, \tag{6}$$

$$F = \begin{bmatrix} 0 & 0 & -V\sin\theta & 0 \\ 0 & 0 & V\cos\theta & 0 \\ 0 & 0 & 0 & \frac{V}{2L\cos^2\psi} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

η – is a vector of noise of measuring.

As discrete analogue of (6), i.e. a relation linking change of errors through a small interval of time d, the following equation (analogue of the equation (3.2) [7]) is accepted

$$\delta\mu_{k+1} = \Phi_k \delta\mu_k + n_k, \tag{7}$$

$$\Phi = I + dF = \begin{bmatrix} 1 & 0 & -dV\sin\theta & 0 \\ 0 & 1 & dV\cos\theta & 0 \\ 0 & 0 & 1 & \frac{dV}{2L\cos^2\psi} \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

n_k - is a vector of random noises. Hereinafter I – is an unit matrix of the corresponding size.

In case of system (2), the analogues of matrices F and Φ, which appear in (6), (7) will be the following matrices

$$F = \begin{bmatrix} 0 & 0 & -\frac{\ell V}{2L} tg\psi\cos\theta - V\sin\theta & -\frac{\ell V}{2L}\frac{\sin\theta}{\cos^2\psi} \\ 0 & 0 & V\cos\theta - \frac{\ell V}{2L} tg\psi\sin\theta & \frac{\ell V}{2L}\frac{\cos\theta}{\cos^2\psi} \\ 0 & 0 & 0 & \frac{V}{2L\cos^2\psi} \\ 0 & 0 & 0 & 0 \end{bmatrix},$$

$$\Phi = \begin{bmatrix} 1 & 0 & -dV\left(\frac{\ell}{2L} tg\psi\cos\theta + \sin\theta\right) & -\frac{dV\ell}{2L}\frac{\sin\theta}{\cos^2\psi} \\ 0 & 1 & dV\left(\cos\theta - \frac{\ell}{2L} tg\psi\sin\theta\right) & \frac{dV\ell}{2L}\frac{\cos\theta}{\cos^2\psi} \\ 0 & 0 & 1 & \frac{dV}{2L\cos^2\psi} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Let on the k step the following process of observations takes place:

$$z_k = H\delta\mu_k + \xi_k, \tag{8}$$

ξ_k - is measurement error.

We concretise this procedure with reference to viewed TR. We will begin with a case, when the GPS receiver is installed in point B (figure 1). In this case it gives the opportunity to obtain (observe) an estimate of co-ordinates and velocity of point B. We will note, that TR is considered as system with nonholonomic constraints. Consequently, velocity of point B is directed along piece AB, i.e. forms the angle θ with the Ox axis. Thus, having the information about projections of velocity of point B on co-ordinate axes, it is possible to gain the estimate of the angle θ. In other words, it is possible to consider, that the GPS receiver which is installed in point B, gives the information about co-ordinates x, y and angle θ.

Let's view more a general case when the GPS receiver is installed in point C. We will guess, that the quantity of ψ in the ratio (3) corresponds to the quantity of ψ, which indicate the data unit of the angle of rotation of a steering wheel. At this guess, it is possible, using the relation (3) to gain the information about angle θ. In other words, and in a case when the GPS receiver is installed in point C, it gives the information about co-ordinates x, y and angle θ. Hence, at the made guess, the GPS receiver does not give the information about magnitude of systematic error $\delta\psi$. Therefore, it is possible to consider, that matrix H in (8) has the following structure:

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}. \tag{9}$$

Thus, using the relations (8), (9) the problem of correction of NS can be stated as a problem of an optimum filtration. It is known (see for example, item 12.4 [1]), that the solution of this problem looks like

$$\begin{aligned} \delta\mu_k &= \delta\bar{\mu}_k + K_k \left(z_k - H\delta\bar{\mu}_k \right), \\ \delta\bar{\mu}_k &= \Phi_k \delta\mu_k. \end{aligned} \tag{10}$$

The matrix of coefficients of amplification of the filter (K_k), which is generating a vector of optimum estimate $\delta\mu_k$, is spotted as follows (the filter equations)

$$K_k = M_k H^T \left(H M_k H^T + R_k \right)^{-1}. \tag{11}$$

$$M_{k+1} = \Phi_k S_k \Phi_k^T + Q_k \tag{12}$$

$$S_k = M_k - K_k \left(H M_k H^T + R_k \right) K_k^T. \tag{13}$$

Here matrices Q_k, R_k are covariance matrices of noises n_k, ξ_k which figure in (7), (8). Matrix M_0 – is covariance matrix of the initial estimate of vector $\delta\mu$. This matrix is given. We will believe, that correction of errors of NS occurs through $j > 1$ steps. In this case, in gaps between the correction moments, change of errors of NS occurs according to the equation (7), and change of their correlation matrix according to (12) (it is possible to believe, that on these steps $H = 0$ i.e., in (11) matrix $K = 0$). On the step on which correction is carried out, the change of a correlation matrix are described by the equations (12), (13).

Correction of systematic error $\delta\psi$ occurs as follows. As a result of correction (10) on a step, when $H \neq 0$, we have the estimate $\delta\psi$ of systematic error $\delta\psi$, (the fourth component of the vector $\delta\mu_k$). This estimate must be subtracted from indications of the angle of rotation of a steering wheel (till to the next moment of correction, in which $H \neq 0$).

So, let t_n is the correction moments (the moment when $H \neq 0$). We will designate $\delta\psi_{n-}$, $\delta\psi_{n+}$ values of the estimate of systematic error $\delta\psi$ before and after t_n. Let's $\delta\tilde{\psi}_n$ is the estimate of a systematic error as result of correction (10) at the moment of t_n. In this case

$$\delta\psi_{n+} = \delta\psi_{n-} - \delta\tilde{\psi}_n. \tag{14}$$

For pinch of accuracy of computing procedures, in such problems are usually used the algorithms, which include the procedures of calculate Cholesky factors of corresponding covariance matrices. Below, the algorithm [5] which is based on QR-decomposition of a matrix will be described. It is supposed, that matrix R_k is invertible. The general case see [5].

5. Computation of Cholesky factors

Let m_k, p_k, q_k, r_k – be Cholesky factors of matrices M_k, S_k, Q_k, R_k accordingly, i.e.

$$M_k = m_k m_k^T,\ \mathrm{S}_k = p_k p_k^T,\ \mathrm{Q}_k = q_k q_k^T,\ \mathrm{R}_k = r_k r_k^T. \tag{15}$$

If the matrix R_k is invertible, the relation (13) can be copied in a following form

$$p_k p_k^T = m_k \left(I + m_k^T H^T R_k^{-1} H m_k \right)^{-1} m_k^T. \tag{16}$$

Let's present the expression standing in brackets, in the form of product of two rectangular matrices

$$\begin{aligned} & I + m_k^T H^T R_k^{-1} H m_k = N_k N_k^T \\ & N_k = \begin{bmatrix} I & m_k^T H^T r_k^{-1} \end{bmatrix}. \end{aligned}$$

By means of orthogonal matrix U, using algorithm of QR-decomposition, we will transform matrix N_k^T as follows

$$\begin{bmatrix} \Lambda_k \\ 0 \end{bmatrix} = U_k N_k^T, \tag{17}$$

where Λ_k – is invertible matrix.

So, according to (16), (17) it is had

$$p_k = m_k \Lambda_k^{-1}. \tag{18}$$

Similarly, we will present the right part of (12) in the form of product of two rectangular matrices. We use QR-decomposition of these matrices which is carried out by orthogonal matrix Z_k.

$$m_{k+1} m_{k+1}^T = T_k T_k^T, \tag{19}$$

$$T_k = [\Phi_k p_k \;\; q_k]\,;$$

$$\begin{bmatrix} X_k^T \\ 0 \end{bmatrix} = Z_k T^T, \tag{20}$$

$$m_{k+1} = X_k. \tag{21}$$

Thus, using m_k, r_k, according to (17), (18), the factor p_k is calculated. Further, using (19 – 21), the factor m_{k+1} is calculated.

6. Example

We will consider TR (figure 1), distance between which axes $|AB| = 2L = 1$m. The GPS receiver installed in point C, $|BC| = \ell = L$. The motion of TR is on a circle. Point B moves with constant velocity V on the circle, radius of which is R. We accept $V = 1$m/s, $R = 10m$. According to the last relation of (1) $tg\psi = \frac{2L}{R} = 10^{-1}$. Measurements occur to frequency 50 Hz, i.e. $d = 0.02s$. In (4), (5) the following hypotheses about measurement errors of the kinematic parameters of TR are accepted: velocity V is registered without noises, measurements of the angle of rotation of steering wheel ψ are accompanied by the additive errors. So the measurement error of angle of rotation $\delta\psi = m + n$ contains systematic error m and casual n which is modelled by sequence of random numbers n_k with a uniform distribution, a zero ensemble average and variance $\sigma = 0.03$. Time interval on which modelling was spent, is accepted equal to 60s. At the initial moment ($t = 0$) position of TR is characterised by angle $\theta = \frac{\pi}{2}$ and co-ordinates of point C: $x = R = 10$m, $y = \ell = 0.5$m. Let's illustrate possibility of use of GPS signals for correction of NS. Frequency of correction is accepted equal to $1\,Hz$. For estimate of angle θ, the relations (3) were used. The error of measurings of projections V_x, V_y – of velocities of point C is accepted equal to 0.1 m/s. The systematic error of measuring of the angle of rotation of a steering wheel is accepted equal to zero ($m = 0$) if $t < 30\ s$. It is supposed, that after the moment of $t = 30\ s$ in the data unit of the angle of rotation of a steering wheel, there is a systematic error equal to 0.2. This error is compensated according to the relation (14). Matrix H is specified by the relation (9). We will spot the Cholesky factors, which are figuring in (15). We suppose, that matrices q_k, r_k do not depend from k, i.e. $q_k = q,\ r_k = r,\ k = 0, 1, \ldots$. We will choose following values for these matrices

$$q = 10^{-2} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0.1 & 0 \\ 0 & 0 & 0 & 0.1 \end{bmatrix}, \quad r = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}.$$

As Cholesky factor (matrix p_0) of covariance matrix of the initial estimate of the vector $\delta\mu$, is accepted the following matrix: $p_0 = 10^{-3}I$. Results of modelling of operation of the featured algorithm of NS, are given on figures 2 - 5.

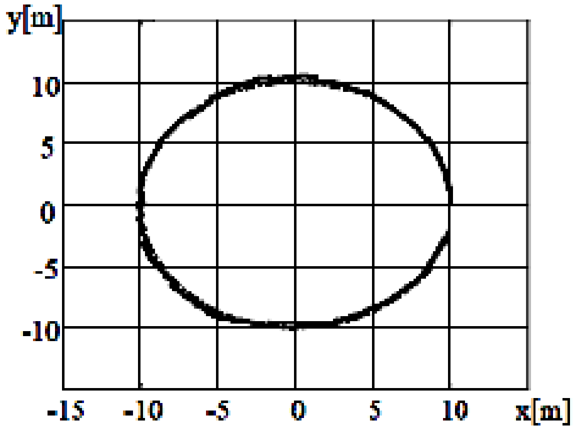

Figure 2. An estimate of trajectory of TR.

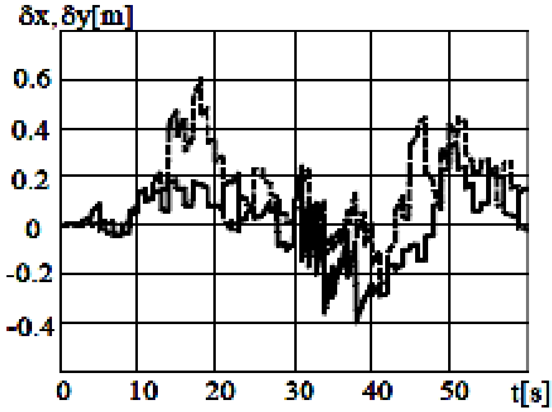

Figure 3. Errors of the estimate of co-ordinates of TR.

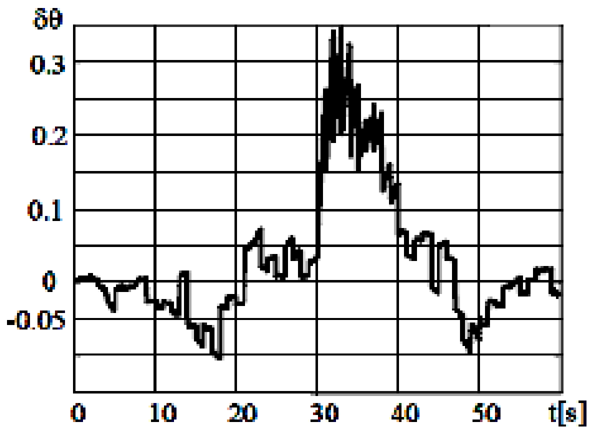

Figure 4. Errors of an estimate of orientation of TR.

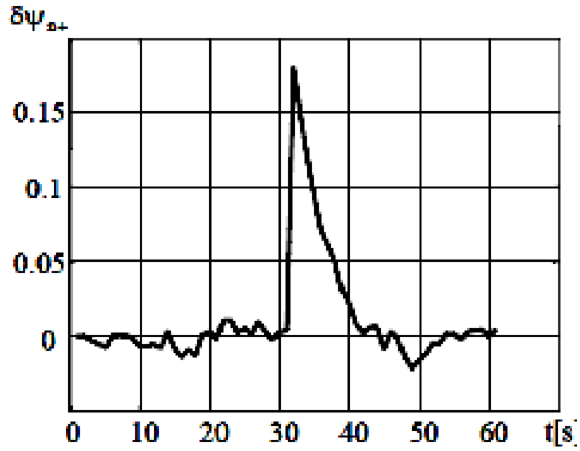

Figure 5. Plot of error of the data unit of an angle of rotation of a steering wheel.

So, on figure 2 the plot of the estimate of the trajectory of point C TR is given (the continuous line corresponds to the gained estimate of a trajectory of point C, the dashed line, corresponds to the initial trajectory). Plot of errors of estimates of co-ordinates are given on the figure 3 (the continuous line corresponds to co-ordinate x, the dashed line - y). Further, on the figure 4 the plot of error of orientation of TR (plot of $\delta\theta$) is shown. The figure 5 gives the representation about dynamics of process of compensation (according to (14)) the systematic error of the data unit of the angle of rotation of a steering wheel. Judging by these plots, the featured algorithm of NS allows to compensate the considerable systematic errors of the data unit.

7. Conclusion

The approach, stated in the paper, is based on the condition that the wheeled robot may be considered as system with nonholonomic constraints. It is shown, that the navigating problem of wheeled TR is possible to solve without use of accelerometers and gyros, i.e. without use INS, but only by use the kinematics parameters of motion of TR (angle of rotation of a steering wheel, etc.). It is assume that speed of back wheels of is registered precisely, and the error of the sensor of the angle of turn of the steering wheel is constant. At these assumptions, using the kinematics parameters of motion as the initial information, the equations of error of the navigating system are received. Correction of navigating parameters and indemnification of error of the sensor of the angle of turn of the steering wheel are fulfilled by use of GPS signals. It is assumed, that GPS receiver may be installed in any point on a line connecting the middle of axes of forward and back wheels. On the example, the efficiency of such navigating system is shown.

References

[1] Bryson, A.E. Jr., Ho-Yu-Chi., (1969), Applied Optimal Control, Optimization, Estimation and Control, Braisdell Publising Company, Massachusetts, Waltham, 544p.

[2] Coopmans, C., (2009), Aggienav: A small, well integrated navigation sensor system for small unmanned aerial vehicles, Proceeding of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference, IDETC/CIE 2009, San Diego, California, USA, pp.1-6.

[3] Coopmans, C., Chaom H., Chen, Y.Q., (2009), Design and implementation of sensing and estimation software in aggienav, a small Uav navigation platform, Proceeding of the ASME 2009 International Design Engineering Technical Conferences & Computers and Information in Engineering Conference. IDETC/CIE 2009, San Diego, California, USA DETC-2009-87675, pp.1-6.

[4] El Najjar, M.E., Bonnifait, Ph., (2005), A Road-matching method for precise vehicle localization using belief theory and Kalman filtering, Autonomous Robots, 19, pp.173-191.

[5] Larin, V.B., (2001), Attutude-determination problems for a rigid body, Int. Appl. Mech., 37(7), pp.870-898.

[6] Larin, V.B., (2014), On navigation of the wheeled transport robot, Journal of Automation and Information Sciences, 46(8), pp.1-9.

[7] Larin, V.B., Tunik, A.A., (2012), On inertial navigation system error correction, Int. Appl. Mech., 48(2), pp.213-223.

[8] Murray, R.M., Sastry, S.S., (1993), Nonholonomic motion planning: steering using sinusoid, IEEE Trans.Automatic Control, 38(5), pp.700-716.

[9] Sukkarieh, S., Nebot, E.M., Durrant-Whyte, H.F., (1999), A high integrity IMU/GPS navigation loop for autonomous land vehicle applications, IEEE Trans. on Robotics and Automat., 15(3), pp.572-578.

[10] Wei, L., Cappelle, C., Ruichek, Y., Zann, F., (2011), GPS and stereovision-Based visual odometry: Application to Urban SceneMapping and Intelligent Vehicle Localization, International Journal of Vehicular Technology, Article ID 439074, pp.1-17.

Vladimir Larin, for a photograph and biography, see TWMS J. Pure Appl. Math., V.2, N.1, 2011, p.160