

## BOOK REVIEW

### **Vibrations of Hollow Elastic Bodies**

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A number of elements of modern technological constructions are thin shells of various shapes and complex structures [1, 4, 6, 8]. The use of shell elements is explained by the requirements to satisfy those concerns which are determined by complicated conditions of various modern space technological constructions, as well as by the possibilities among their rational parameters choice as for strength and reliability under various operating conditions. Apart from static forces deformable elastic bodies can be subjected to the action of various kinds of time-varying dynamic loads. Among the dynamical processes, it is especially important to single out oscillatory harmonic processes in which the external and generated stresses and displacements of the body vary periodically in time. The solution of the problems of the vibrations of the elastic bodies is connected with great difficulties. Some numerical and analytical approaches for the solution of such problems are given in [1-5, 7, 9, 10]. One of the main characteristics of any periodic harmonic process is the vibrations frequency, defined as a number of periodic motions per unit time. An important feature of harmonic vibrations is that the deformation response of a body to a periodic external action essentially depends on the frequency of the change in external forces. Under certain excitation frequencies, called eigenfrequency or resonant frequencies, the body vibrations amplitudes increase significantly (the phenomenon of resonance), which due to the appearance of large alternating stresses in absolute magnitude, can lead to a decrease in strength, cracking, or even destruction of the construction. Natural vibrations of elastic bodies is determined by the corresponding frequencies as well as by the so-called vibration modes or the modes which are characterized by the normalized amplitudes of the displacements of the points of the body. Both the forms of natural vibrations and frequencies do not depend on the excitation method and the amplitudes of the provoking vibrations and are determined by the stiffness and mass characteristics of the body, and by the ways of its fixation. An important factor in the process of mechanical behavior of shell structural elements investigation for various purposes to provide strength and carrying capacity within the operation, is the information about dynamic characteristics, in particular, resonance regimes.

The reviewed book is dedicated to the study of forced and natural vibrations of a wide class of shell structures based on 3D elasticity theory. The problems of stress-strain state of a truncated hollow cone were considered too. The main feature of this book is the application for the solution of the difficult dynamic and static problems of shell and elasticity theories of analytical approach, basing on asymptotic methods. The book consists of four chapters and the appendix.

**The first chapter** considers forced vibration of an isotropic hollow cylinders with different boundary conditions under the action axisymmetric loads based on 3D elasticity theory. The method of homogeneous solution was used there. And there presented the asymptotic construction of homogeneous solutions corresponding to different groups of dispersion equation. The mixed boundary value problem for the forced harmonic vibrations of an elastic band was exactly solved. The asymptotic method to remove the differential equation on the reduction of

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the three-dimensional differential equations to a two dimensional is obtained for each characteristic of stress-strain state in the axisymmetric case. The torsional vibration of an isotropic hollow cylinders have been studied too. The asymptotic behavior of the dynamic problem of the elasticity theory is investigated for a hollow cylinder with a rigid sealing of the side surface and with the mixed boundary conditions on the side surface based on homogeneous solution.

**The second chapter** contains the results obtained for the dynamics problems of the spherical layers based on three dimensional elasticity theory. In case of axisymmetric vibrations of the hollow sphere the homogeneous solution are constructed. An asymptotic analysis of the homogeneous solution for a hollow sphere corresponding to different groups of roots of the dispersion equation were performed. The presented method of constructing the homogeneous solution is rather versatile and is not dependent of different parameters of the sphere including its thickness. A generalized conditions of orthogonality of systems of the homogeneous solution is proved. One class of the boundary conditions on the side surface, admitting an exact solution of the problem of the forced vibrations of hollow sphere is identified. The problem of the torsional vibrations of a spherical layers is solved analytically. Non-axisymmetric the forced vibration of the spherical layers based on three dimensional elasticity theory are considered also.

**The third chapter** is dedicated to a very important problem of determining the vibration frequencies of the vibrations of finite elasticity solids and comparison of the results with those obtained by the different applied theories. The construction of an asymptotic process for finding frequencies of free axisymmetric cylinder and closed sphere are presented. The asymptotic process in details for a cylinder with free side surface and simple supported at the ends and a closed a hollow sphere with free face surface are considered. The author's approaches allow to devise algorithms that are able to capture all free frequencies in a given interval which undoubtedly represents certain scientific and practical value. The frequencies of the thickness vibrations of cylindrical and spherical elastic bodies are determined.

**The fourth chapter** is dedicated to the investigation of the stress-strain state of the hollow conical bodies and the thin plates based on the theory of the elasticity. An asymptotic method of integrating boundary value problem for the three-dimensional equations for analysis the stress-strain state for corresponding of the elasticity bodies is developed. The hollow conical bodies and the thin plates whose thickness varies linearly along the generatrix are considered. Proceeding from Papkovitch-Neuber general solution, a stress-strain state a thin plate with variable thickness subject to the action of non- axisymmetric loads are studies. The analysis of the stress-strain state behavior of the conical bodies shows, that in case of the cylindrical and spherical bodies, is composed of three types: internal stress state, the simple end effect and the boundary layer state.

**In the appendix**, a solution of the axisymmetric problems on stress concentration around a circular hole in the plate with the boundary is load with normal forms is presented. The numerical results for the solution of this problem are presented. This problem can be regarded as a model for the corresponding problems in shell theory. The exact solution of the problem of the axisymmetric vibrations of a thick cylindrical shell under given mixed boundary conditions at the ends by means of homogeneous solution is derived. The numerical analysis for the parabolic distribution of the normal stress on the end surface and at the zero radial displacement is carried out. I am convinced that the book will be of great interest to many graduate students and to researchers dealing with various aspects of solid mechanics.

The scientific results of the author obtained here are important for further fundamental investigations concerned with dynamics of solids; they should also be effective while solving many practical problems. In my opinion, the monograph "Vibration of Hollow Elasticity Bodies" by Magomed F. Mekhtiev constitutes a significant contribution that facilitates analysis of processes associated with vibrations of elastic solids based on the 3D elasticity theory.

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