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IMPACT OF ADVERTISEMENT ON RETAILER'S INVENTORY WITH NON-INSTANTANEOUS DETERIORATION UNDER PRICE-SENSITIVE QUADRATIC DEMAND

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Abstract. This paper deals with an inventory system of dynamic pricing and stock control of non-instantaneous deteriorating items. To hold dynamic nature of the problem, the price is modeled as a time-dependent function of the initial selling price and the discount rate. Demand depends on price and advertisement. The product is sold at the initial price value in the time period with no deterioration; afterward, it is exponentially discounted to boost customers demand. Inspired by the significant impact of promotion on encouraging sales, the demand rate is linked to the frequency of advertisement in each cycle time as well. The retailer invests investments on advertisement and preservation technology to preserve the item. The objective is to optimize the total profit of the retailer with respect to advertisement, selling price, cycle time and investment for preservation technology. Effect of advertisement is analyzed. If the retailer use preservation technology and advertisement then he can earns more profit. The model is supported with numerical examples and also established best scenario of the model. Sensitivity analysis is done to deduce managerial insights.

Keywords: dynamic selling price, non-instantaneous deterioration rate, effect of advertisement, preservation technology investment.

AMS Subject Classification: 90B05.

1. Introduction

To increase the demand and attract more customer retailer should do publicity of his product by any advertisement tools. It is well established that there is a negative relationship between demand and the price of a product (Avinadav et al.[1]) and this relationship can be modeled in a diverse variety of ways. Despite the mentioned diversity, these demand models fall into two main categories named additive and multiplicative demand models.Wu et al. [32] was the first research to assume non-instantaneous deterioration which was appropriate for deterioration pattern of many products. Kocabiyikoglu and Popescu [12] have provided some examples of common demand models in this area. Soni and Patel [27] examined optimal pricing and inventory policies for non-instantaneous deteriorating items as well. This model was extended by Soni and Patel [28] by considering imprecise deterioration free time and credibility constraint. Cai et al. [3] proposed one of the rare studies on dynamic pricing which modeled price as a function of time. Optimal policy was obtained by considering feedback of price on demand per time unit. Wang et al. [30] also considered price as a function of time and modeled a non-instantaneous deterioration pattern.

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1.1. **Literature review on Promotional tool.** Advertising has served a critical determination in the business world by permitting sellers to effectively compete with one another for the attention of buyers. Whether the goods and services your business provides are a requirement, a luxury or just a bit of whimsy, you can't rely on a one-time declaration or word-of-mouth conversation to keep a steady stream of customers. A strong promise to advertise is as much an external call to action as it is an internal strengthening to your sales team. Tsao and Sheen [29], Shah et al. [24] discussed the dependency of demand on advertisement. Barrn and Sana [4] studied multi-item EOQ inventory model in a two-layer supply chain while demand varies with a promotional effort. Also, Rabbani et al. [13] studied coordinated replenishment and marketing policies for non-instantaneous stock deterioration problem. Demand was modeled as a linear function of price, exponential function of time and quadratic function of advertisement cost.

1.2. **Literature review on deterioration.** Due to drastic environmental changes, most of the items losses its efficiency over time, termed as deterioration. Deterioration of goods likes, explosive liquids, fruits, root vegetable, radioactive substances, medicine, blood etc. Out of several studies on deteriorating items only few of them have considered fixed Life-time issue of deteriorating items. Ghare and Schrader [8] considered effect of deterioration in inventory model. The criticise articles by Raafat [15], Shah and Shah [26], Goyal and Giri [9], Bakker et al. [2], on deteriorating items for inventory system throw light on the role of deterioration. Sarkar [17] developed two-level trade credit policy with time varying deterioration rate and time dependent demand. Chung and Cardenas-Barrn [5] modelled simple algorithm for deteriorating items under stock-dependent demand and two-level trade credit in a supply chain comprising of three players. Some motivating articles are by Ouyang et al. [14], Sarkar et al. [18], Chung et al. [6], Wu et al. [31], Sarkar and Saren [16] and their cited references. Furthermore, Shah and Barrn [24] deliberated retailer's decision for ordering and credit policies for deteriorating items when a supplier offers order-linked credit period or cash discount. Sarkar et al. [20] established an inventory model with trade-credit policy and variable deterioration for fixed lifetime products. Sarkar and Saren [19] established a partial trade-credit policy of retailer with exponentially deteriorating items.

1.3. **Literature review on Preservation Technology.** On the other hand, to reduce the deterioration, Hsu et al. [11] developed an inventory model with preservation technology investment to minimize the deterioration rate of inventory for constant demand. Dye and Hsieh [7] analyzed an optimal replenishment policy for deteriorating items with effective investment in preservation technology. Hsieh and Dye [10] examined a production inventory model incorporating the effect of preservation technology investment when demand is fluctuating with time. Recently, Shah and Shah [26] evaluated an inventory model for optimal cycle time and preservation technology investment for deteriorating items with price-sensitive stock-dependent demand under inflation. Later on Shah et al. [23] estimated optimal policies for deteriorating items with maximum lifetime and two-level trade credits. Moreover, Shah et al. [25] established optimal policies for time-varying deteriorating item with preservation technology under selling price and trade credit dependent quadratic demand in a supply chain.

In this paper, retailers best policies are analyzed under advertisement, time and price- sensitive demand with constant deterioration. The retailer invest capital on advertisement and preservation technology. To reduce deterioration, preservation technology is incorporated and analyzed effect of advertisement and preservation on retailers profit. Under above assumptions, the objective is to maximize the profit of retailer with respect to the advertisement cycle time, selling price and investment of preservation technology.

The rest of the paper is organized as follow. Section 2 presents the notations and the assumptions that are used. Section 3 derives the mathematical model of the inventory problem. Section 4 establishes the proposed inventory model with numerical examples. This section also provides some managerial insights. Finally, Section 5 provides conclusion and future research directions.

2. Notation and assumptions

The proposed inventory problem is based on the following notation and assumptions.

2.1. Notation.

Table 1.

Retailer's parameters:	
A_r	fixed production setup cost per order (\$/order)
$HC_r(t)$	time dependent holding cost (\$/unit / unit time)
h_r	holding cost rate per unit per annum
p_0	initial price of the product (\$/unit / unit time)
$p(t)$	the dynamic price of product per unit at any time t(\$/unit) (a decision variable)
C_r	unit purchase price per item (\$/unit)
C_a	cost of each advertisement ((\$/lot)
C_d	the deterioration cost per unit
A	the frequency of advertisement in each cycle (lot/week) (a decision variable)
T	cycle time (unit time) of the retailer (a decision variable)
$T_d = \nu T$	the length of deterioration free time (unit time)
ν	rate of delay period
θ	constant Deterioration rate; $0 \leq \theta \leq 1$
$I_{r1}(t)$	inventory level at any time t(units) during$0 \leq t \leq T_d$
$I_{r2}(t)$	inventory level at any time t(units) during$T_d \leq t \leq T$
$I_{\max}$	maximum inventory level
SR	selling Revenue
ω	price sensitive factor
ε	sensitivity factor of changes in price
σ	discount variable for each time unit passing after the start of deterioration
u	preservation technology investment per unit time (in \$)(decision variable)
$f(u)$	$= 1 - \frac{1}{1+\mu u}$ proportion of reduced deterioration item (in year), $\mu > 0$
μ	rate of preservation
$\pi_r(A, p, T, u)$	total Profit per unit time of the retailer inventory system (\$/unit time)

Relations between parameters:

(1) $p_0 > C_r$
(2) $0 \leq \theta < 1$.

Parameters of Retailer:

$R(A,p,t)$	Advertisement, selling price and time dependent quadratic demand rate; $R(A,p,t) = a \cdot \left(1 + bt - ct^2 - \omega p(t) - \varepsilon p'(t)\right)(1+A)^{\lambda}$, where $a > 0$ is scale demand, $0 < b, c < 1$ are rates of change of demand.
$\pi_r(A,p,T,u)$	Total profit of the retailer.

2.2. **Assumptions.**

(1) The inventory system involves single non-instantaneous deteriorating item.

(2) The demand rate is a function of the selling price, frequency of advertisement and changes in price per time unit and time dependent quadratic demand. The demand rate $R(A,p,t) = a \cdot \left(1 + bt - ct^2 - \omega p(t) - \varepsilon p'(t)\right)(1+A)^{\lambda}$ (say) is function of time, advertisement and selling price, $a > 0$ is total market potential demand, $0 \leq b < 1$ denotes the linear rate of change of demand with respect to time, $0 \leq c < 1$ denotes the quadratic rate of change of demand, λ is the shape parameter of the advertisement. As in Shah *et al.* (2013)$0 \leq \lambda < 1$, the logic behind this is that, the demand rate is an increasing function of the frequency of advertisement ($\lambda \geq 0$) and also large jumps in demand rate by single increase of the frequency of advertisement is not rational$\lambda < 1$.

(3) As the deterioration starts afterT_d, the selling price tends to increase the inventory depletion rate. Due to computational controllability and dynamic price of the product decreases exponentially over time and it is formulated as $p(t) = \begin{cases} p_0 & 0 \leq t \leq T_d \\ p_0 e^{-\sigma(t-T_d)} & T_d \leq t \leq T \end{cases}$ where, $p_0$is the initial price and $\sigma$is of discount variable for each time unit passing after the start of deterioration.

(4) Time horizon is infinite.

(5) Shortages are not allowed.

(6) Lead time is zero or negligible.

(7) The non-instantaneous rate of constant deterioration isθ at any time$t \geq T_d$, where $0 \leq \theta \leq 1$.

(8) The proportion of reduced deterioration rate,$f(u)$, is assumed to be a continuous increasing and concave function of investment u on preservation technology ,i.e. $f'(u) > 0$ and $f''(u) > 0$. WLOG, assume $f(0) = 0$.

In the next section, the proposed inventory model for the retailer is developed.

3. Mathematical model

In this section, according to above assumption the joint dynamic pricing and inventory control of non-instantaneous deteriorating item is modelled. As the above assumption demand is price sensitive and by starting deterioration of the inventory, the price is exponentially decrease in order to increase the inventory reduction rate:

$$p(t) = \begin{cases} p_0 & 0 \leq t \leq T_d \\ p_0 e^{-\sigma(t-T_d)} & T_d \leq t \leq T. \end{cases}$$

Therefore, the changes in price per time unit are defined as:

$$p'(t) = \begin{cases} 0 & 0 \leq t \leq T_d \\ -\sigma p_0 e^{-\sigma(t-T_d)} & T_d \leq t \leq T. \end{cases}$$

In this inventory system initially I_0units of items arrive at the beginning of each cycle. Based on the value of $T$and T_d two scenarios are arises: Scenario 1: $T_d \leq T$ and Scenario 2: $T_d \geq T$. Let us discuss two scenarios in details as follow.

Scenario 1: $T_d \leq T$.

In this scenario during time interval $[0\ ,\ T_d]$, the inventory system exhibits no deterioration and the inventory level decreases due to demand only. Consequently the inventory level declines due to demand and deterioration during time interval $[T_d\, ,\, T]$. Finally the replenishment cycle discharges as the inventory level reaches zero.

So, during time interval $[0\ ,\ T_d]$ the changes of the inventory level per time unit is represented by the following differential equations:

The retailer's inventory level at time t during a cycle of length $T$is given by

$$\frac{dI_{r1}(t)}{dt} = -R(A, p, T),\, 0 \leq t \leq T_d$$

$$\frac{dI_{r2}(t)}{dt} = -\theta(1 - f(u))\, I_{r2}(t) - R(A, p, T),\, T_d \leq t \leq T.$$

With boundary conditions $I_{r1}(0) = I_0$ and $I_{r2}(T) = 0$.

Solving above differential equations we get,

$I_{r1}(t) = -a(1 + A)^{\lambda}\left(t + \frac{bt^2}{2} - \frac{ct^3}{3} - \omega p_0 t\right) + I_0$ and

$$I_{r2}(t) = e^{\theta(-1+f(u))t}$$

$$\times \left(\begin{array}{l} -a(1+A)^{\lambda} \left(\begin{array}{l} e^{-\theta(-1+f(u))t} \left(\begin{array}{l} \frac{1}{\theta(-1+f(u))} \\ -\frac{b(-\theta(-1+f(u))t-1)}{\theta^2(-1+f(u))^2} \\ -\frac{1}{\theta^3(-1+f(u))^3} \\ \left(c \left(\begin{array}{l} \theta^2(-1+f(u))^2 t^2 \\ +2\theta(-1+f(u))t+2 \end{array} \right) \right) \end{array} \right) \\ +\frac{\omega p_0 e^{t\theta - t\theta f(u) - \sigma t + \sigma T_d}}{\theta - \theta f(u) - \sigma} - \frac{\varepsilon p_0 e^{t\theta - t\theta f(u) - \sigma t + \sigma T_d}}{\theta - \theta f(u) - \sigma} \end{array} \right) \\ +a(1+A)^{\lambda} \left(\begin{array}{l} e^{-\theta(-1+f(u))T} \left(\begin{array}{l} \frac{1}{\theta(-1+f(u))} - \\ \frac{b(-\theta(-1+f(u))T-1)}{\theta^2(-1+f(u))^2} \\ -\frac{1}{\theta^3(-1+f(u))^3} \\ \left(c \left(\begin{array}{l} \theta^2(-1+f(u))^2 T^2 \\ +2\theta(-1+f(u))T+2 \end{array} \right) \right) \end{array} \right) \\ +\frac{\omega p_0 e^{T\theta - T\theta f(u) - \sigma T + \sigma T_d}}{\theta - \theta f(u) - \sigma} - \frac{\varepsilon p_0 e^{T\theta - T\theta f(u) - \sigma T + \sigma T_d}}{\theta - \theta f(u) - \sigma} \end{array} \right) \end{array} \right).$$

It is cleared that, $I_{r1}(T_d) = I_{r2}(T_d)$. Then the maximum inventory level

$$I_0 = I_{\max} = e^{\theta(-1+f(u))T_d}$$

$$\times\left(\begin{array}{l} -a\left(1+A\right)^{\lambda}\left(\begin{array}{l} e^{-\theta(-1+f(u))T_d}\left(\begin{array}{l} \frac{1}{\theta(-1+f(u))} \\ -\frac{b(-\theta(-1+f(u))T_d-1)}{\theta^2(-1+f(u))^2} \\ -\frac{1}{\theta^3(-1+f(u))^3} \\ \left(c\left(\begin{array}{l}\theta^2\left(-1+f\left(u\right)\right)^2{T_d}^2 \\ +2\theta\left(-1+f(u)\right)T_d+2\end{array}\right)\right)\end{array}\right) \\ +\frac{\omega p_0 e^{T_d\theta-T_d\theta f(u)}}{\theta-\theta f(u)-\sigma}-\frac{\varepsilon p_0 e^{T_d\theta-T_d\theta f(u)}}{\theta-\theta f(u)-\sigma}\end{array}\right) \\ +a\left(1+A\right)^{\lambda}\left(\begin{array}{l} e^{-\theta(-1+f(u))T}\left(\begin{array}{l} \frac{1}{\theta(-1+f(u))} \\ -\frac{b(-\theta(-1+f(u))T-1)}{\theta^2(-1+f(u))^2} \\ -\frac{1}{\theta^3(-1+f(u))^3} \\ \left(c\left(\begin{array}{l}\theta^2\left(-1+f\left(u\right)\right)^2T^2 \\ +2\theta\left(-1+f(u)\right)T+2\end{array}\right)\right)\end{array}\right) \\ +\frac{\omega p_0 e^{T\theta-T\theta f(u)-\sigma T+\sigma T_d}}{\theta-\theta f(u)-\sigma}-\frac{\varepsilon p_0 e^{T\theta-T\theta f(u)-\sigma T+\sigma T_d}}{\theta-\theta f(u)-\sigma}\end{array}\right)\end{array}\right)$$

$$+a(1+A)^{\lambda}(T_d+\frac{b{T_d}^2}{2}-\frac{c{T_d}^3}{3}-\omega p_0 T_d).$$

Therefore, the order quantity is equal to$Q=I_0=I_{\max}$.
Now, the retailer's sales revenue per cycle time T is

$$SR=\int_o^T p\left(t\right)\ R\left(A,u,T\right)\ dt$$

Now, the total cost per unit time of retailer is comprised by

- Ordering cost per unit:$OC_r=A_r$
- Purchase cost per unit :$PC_r=C_r\cdot I_0$
- Inventory holding cost per unit:$HC_r=h_r\left[\int_0^{T_d} I_{r1}(t)\ dt+\int_{T_d}^{T} I_{r2}(t)\ dt\right]$
- Advertisement cost per lot :$AC_r=C_a\cdot A$
- Deterioration cost:$DC_r=C_d\cdot\int_{T_d}^{T}\theta\, I_{r2}(t)\ dt$
- Investment for Preservation Technology:$PTI=u\ T$.

Hence, the profit per unit time of retailer for scenario $T_d\le T$ is

$$\pi_{r1}\left(A,p,T,u\right)=\frac{1}{T}\left(SR-OC_r-PC_r-HC_r-AC_r-DC_r-PTI\right)$$

Scenario 2: $T_d\ge T$.

In this case no deterioration occurs in the inventory system so, no need for preservation technology investment. From the above mentioned sales revenue and cost parameters, the profit per unit time of retailer for this scenario $T_d\ge T$ is

$$\pi_{r2}\left(A,p,T,u\right)=\frac{1}{T}\left(SR-OC_r-PC_r-HC_r-AC_r\right).$$

Therefore, the total profit per unit time of the retailer is

$$\pi_r\left(A,p,T,u\right)=\left\{\begin{array}{ll}\pi_{r1}\left(A,p,T,u\right), & T_d\le T\\ \pi_{r2}\left(A,p,T,u\right), & T_d\ge T.\end{array}\right.$$

It should be cleared that the profit function $\pi_r(A,p,T,u)$ is continuous at $T = T_d$. The profit function $\pi_r(A,p,T,u)$ is a continuous function of Advertisement A, selling price p, cycle time T and investment of preservation technology u. We will establish validation of the proposed model using numerical examples. The maximization of the total profit will be shown graphically for the obtained results.

4. Numerical example and sensitivity analysis

4.1. **Numerical example. Example 1. (For Scenario 1):** Consider $a = 600$ units, $b = 0.3$, $c = 0.2$, $\theta = 0.1$ (10%), $\varepsilon = 0.04$, $\omega = 0.05$, $\lambda = 0.1$, $\mu = 1.3$, $\sigma = 0.8$, $A_r = \$250$, $h_r = \$0.4$ per unit per cycle, $C_r = \$4$ per unit, $C_a = \$80$ per lot, $C_d = \$1$ per unit and $\nu = 0.8$. The optimal values of the decision variables are advertisement $A = 4.14 \approx 4$ times per cycle, cycle time $T = 1.3291$ weeks, selling price $p_0 = \$14.48$ and $u = \$1.56$. This results retailer's profit as **$ 2658.74**.

The concavity of the profit function is obtained by the well-known Hessian matrix. Now, Hessian matrix is for the above inventory system is

$$H(A,p,T,u) = \begin{pmatrix} \frac{\partial^2\pi_r(A,p,T,u)}{\partial A^2} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial A\partial p} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial A\partial T} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial A\partial u} \\ \frac{\partial^2\pi_r(A,p,T,u)}{\partial p\partial A} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial p^2} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial p\partial T} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial p\partial u} \\ \frac{\partial^2\pi_r(A,p,T,u)}{\partial T\partial A} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial T\partial p} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial T^2} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial T\partial u} \\ \frac{\partial^2\pi_r(A,p,T,u)}{\partial u\partial A} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial u\partial p} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial u\partial T} & \frac{\partial^2\pi_r(A,p,T,u)}{\partial u^2} \end{pmatrix}.$$

Using the above Example 1, we get the hessian matrix $H(A,p,T,u)$ at the point (A,p,T,u)

$$H(A,p,T,u) = \begin{pmatrix} -11 & 0 & 39 & 0 \\ 0 & -61 & -14 & 0 \\ 39 & -41 & -1301 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$

As in Barrón and Sana (2015), if the eigenvalues of the Hessian matrix at the solution (A,p,T,u) are all negative, then the profit function $\pi_r(A,p,T,u)$ is maximum at that solution. Here, Eigenvalues of above Hessian matrix are $\lambda_1 = -1301.9$, $\lambda_2 = -60.545$, $\lambda_3 = -9.349$, $\lambda_4 = -1.022$. Also, determinant is positive (i.e. det (H) =777363). So, the profit function $\pi_r(A,p,T,u)$ is maximum.

Example 2. (For Scenario 2): Taking same data as given in example 1 except $\lambda = 0.05$, $\nu = 1.2$, $C_d = \$0$ per unit and $u = 0$,the optimal value of the decision variables are advertisement $A = 1.46$ times per cycle, cycle time $T = 3.73$ weeks, selling price $p_0 = \$09.13$. This results retailer's profit as **$ 955.68.**

Total profit for the above two different scenarios can be described by following bar graph Fig.4. The optimum solution is exhibited in Table 1.

Table 2. Optimal Solution.

Scenario	Total Profit ($)	Decision (per lot, in weeks & in $)
$T_d \le T$	**2658.74**	$A = 4.147$ $p_0 = 14.48$ $T = 1.329$ $u = 1.563$
$T_d \ge T$	955.68	$A = 1.46$ $p_0 = 9.13$ $T = 3.73$

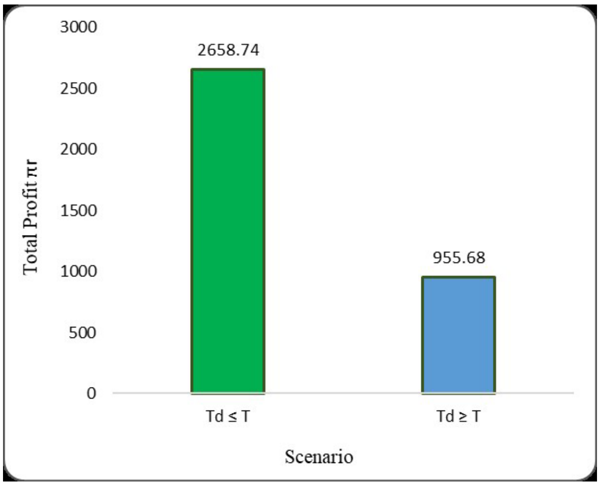

Figure 1. Optimal Solution.

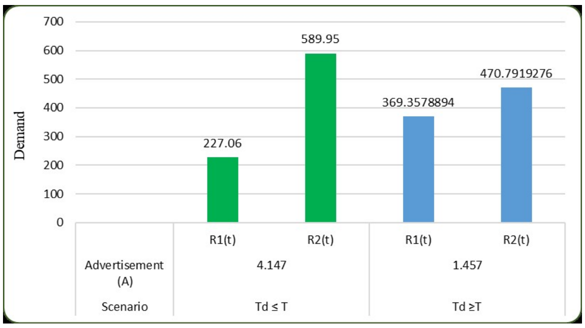

Figure 2. Effect of advertisement on demand.

From the Example 1, Fig. 1, Fig. 2 and Table 1, it is clear that the scenario1 (i.e.$T_d \le T$) is the best case for this model. If the retailer use preservation technology and advertisement then he can earns more profit. It is shown in the Figure 2 due to advertisement demand will increase. When product will start deteriorate, if the retailer spend money on advertisement and preservation technology and also decreases selling price then obviously demand will boost. Hence, he will earn more profit compare to Scenario 2 (i.e.$T_d \ge T$).

4.2. **Sensitivity analysis for the inventory parameters.** Therefor for the different inventory parameter, the sensitivity analysis of example 1 is carried out by changing one variable at a time as−20%,−10%,10% and 20%.The variations in Advertisement are presented in Fig. 3.

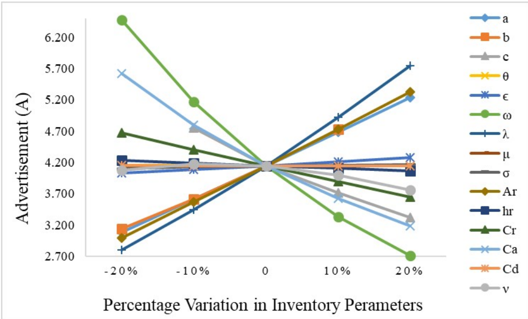

Figure 3. Variations in advertisement (A).

In Fig.3, Advertisement is plotted for variations in inventory parameters. Scale demand, linear rate of demand, shape parameter of the advertisement and ordering cost increases advertisement rapidly whereas quadratic rate of change of demand, price sensitive factor, purchase cost and rate of delay period decreases advertisement rapidly. Moreover, Sensitivity factor of change in price and discount variable for each time unit passing after the start of deterioration increases advertisement slowly however rate of holding cost decreases advertisement slowly. In addition, no effect on advertisement to change deterioration rate, rate of preservation and deterioration cost.

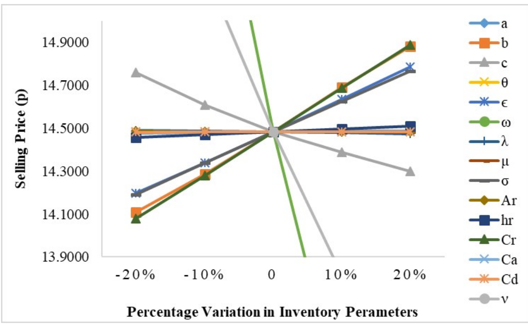

Figure 4. Variations in selling price (p).

In Fig.4, Selling price is plotted for variations in inventory parameters. Sensitivity factor of change in price and discount variable for each time unit passing after the start of deterioration increases selling price rapidly whereas price sensitive factor and rate of delay period decreases selling price rapidly. Moreover, Scale demand, quadratic rate of change of demand, shape parameter of the advertisement and ordering cost decreases selling price slowly however linear rate of demand, rate of holding cost, purchase cost, advertisement cost and deterioration cost increases selling price slowly. In addition, change in rate of preservation selling price remains constant.

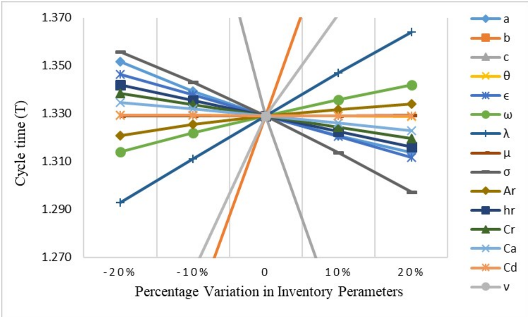

Figure 5. Variations in cycle time (T).

In Fig.5, cycle time is plotted for variations in inventory parameters. Scale demand, Sensitivity factor of change in price, rate of holding cost, purchase cost, advertisement cost and discount variable for each time unit passing after the start of deterioration decreases cycle time slowly however ordering cost increases cycle time slowly. Moreover, linear rate of demand, price sensitive factor, shape parameter of the advertisement and rate of delay period increases cycle time rapidly whereas quadratic rate of change of demand decreases cycle time rapidly. In addition, cycle time remain constant when changes in deterioration rate, rate of preservation and deterioration cost.

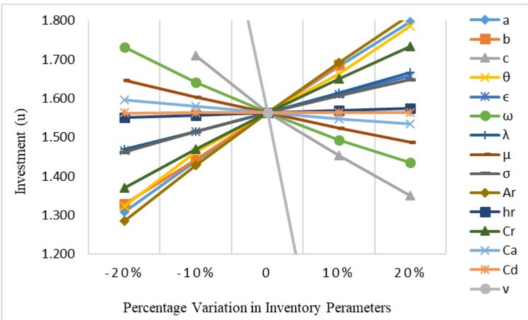

Figure 6. Variations in preservation technology investment (u).

In Fig.6, preservation technology investment is plotted for variations in inventory parameters. Scale demand, linear rate of demand, deterioration rate, shape parameter of the advertisement, ordering cost and purchase cost increases preservation technology investment rapidly whereas quadratic rate of change of demand, price sensitive factor and rate of delay period decreases preservation technology investment rapidly. Moreover, Sensitivity factor of change in price, discount variable for each time unit passing after the start of deterioration and rate of holding cost increases preservation technology investment slowly however, rate of preservation and advertisement cost decreases preservation technology investment slowly. In addition, preservation technology investment remain constant when changes in and deterioration cost.

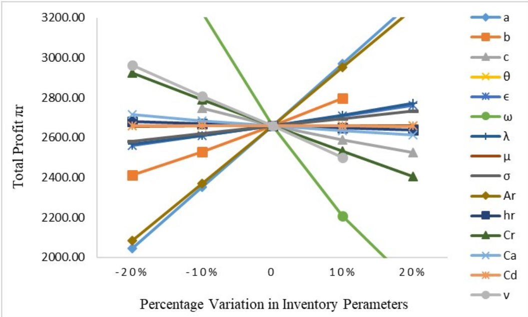

Figure 7. Variations in total profit (π).

In Fig.7, total profit is plotted for variations in inventory parameters. Scale demand, linear rate of demand and ordering cost increases total profit rapidly whereas price sensitivity factor, purchase cost rate and rate of delay period decreases total profit rapidly. Moreover, Sensitivity factor of change in price, shape parameter of the advertisement, rate of preservation and discount variable for each time unit passing after the start of deterioration increase total profit slowly however quadratic rate of change of demand, deterioration rate, holding cost rate, advertisement cost and deterioration cost rate decreases total profit slowly.

5. Conclusion

In this paper, we consider a retailer's model for constant deteriorating item under preservation technology and advertisement investment, with selling price and time dependent demand. To reduce non instantaneous deteriorating items retailer invest money on advertisement to increase demand and preservation. The total profit of the retailer with respect to advertisement, cycle time, selling price and preservation investment is maximized.

If the retailer use preservation technology and advertisement then he can earns more profit. We can easily analyzed from the model that if retailer uses advertisement, demand will increase. When product will start deteriorate, if the retailer spend money on advertisement and preservation technology and also decreases selling price then obviously demand will boost.The decision policies are analyzed for the decision maker. For numerical examples, retailer reaches the maximum profit and carry-out sensitivity analysis with respect to inventory parameters. Current research has several possible extensions. For example, the model can be further generalized by allowing shortages and taking more items at a time. One can also analyze the Multi layered supply chain.

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